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Applied Analysis Tutorials

Sheet: 5

Submission: Tuesday, 17.11.2009 during the next tutorial class.

1. Let μ be an outer measure on Ω. Show that the following hold true:
   (a) Every set $A \subset \Omega$ with $\mu(A) = 0$ is μ-measurable.
   (b) If $A \subset \Omega$ is μ-measurable then A^c is also μ-measurable.
   (c) If $A \subset \Omega$ is μ-measurable and $B \subset \Omega$ then A is μ_B-measurable.
2. Let Ω be a set and μ be an outer measure on Ω. If $A_1, A_2 \subset \Omega$ are μ-measurable then the intersection $A_1 \cap A_2$ is also μ-measurable.
3. Let μ be an outer measure on a set X.
   (a) If A is a null set, then show that
   $$\mu(B) = \mu(A \cup B) = \mu(B \backslash A)$$
   holds for every subset B of X.
   (b) If E is a measurable subset of X, then show that for every subset B of X the following equality holds
   $$\mu(E \cup B) + \mu(E \cap B) = \mu(E) + \mu(B).$$
4. Let μ be an outer measure on Ω. If a sequence $\{A_n\}$ of subsets of X satisfies $\sum_{n=1}^{\infty} \mu(A_n) < \infty$, then show that the set
   $$D = \{x \in X : x \text{ belongs to } A_n \text{ for infinitely many } n\}$$
   is a null set.