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WS 2009/2010

Applied Analysis Tutorials

Sheet:7

Submission: Tuesday, 01.12.2009 during the next tutorial class.

1. Let $\Omega \subset \mathbb{R}^n$ be a set, $\mathcal{B}(\Omega)$ be the Borel σ-algebra on Ω and let $x_0 \in \Omega$. For $A \in \mathcal{B}(\Omega)$ we define

$$\delta_{x_0} := \begin{cases} 1 & \text{if } x_0 \in A \\ 0 & \text{else} \end{cases}$$

   Verify that δ_{x_0} is a measure on $(\Omega, \mathcal{B}(\Omega))$.

2. (a) Let $(\Omega, \mathcal{A}, \nu)$ be a measure space and let $B \in \mathcal{A}$. Show that $(\Omega, \mathcal{A}, \nu_B)$ is a measure space.
   (b) Let $\nu_1, \nu_2, ..., \nu_n$ be measures on a fixed measurable space $(\Omega, \mathcal{A})$ and let $\lambda_1, ..., \lambda_n \in \mathbb{R}^+$. Show that $\nu := \sum_{k=1}^{n} \lambda_k \nu_k$ is a measure on $(\Omega, \mathcal{A})$ where

$$\nu(A) := \nu := \sum_{k=1}^{n} \lambda_k \nu_k(A), A \in \mathcal{A}.$$

3. Let $(\Omega, \mathcal{A})$ be a measurable space. Show that
   (a) Every constant function is measurable.
   (b) Let $A \subset \Omega$. Then A is $\mathcal{A}$-measurable iff χ_A is $\mathcal{A}$-measurable.

4. (a) Let $A \subset \mathbb{R}^n, x \in \mathbb{R}^n$ and $\alpha > 0$. Show that A is Lebesque-measurable iff $x + \alpha A$ is Lebesque-measurable.
   (b) Show that in order to prove that every left-open cell Q is Lebesque-measurable it suffices to consider $Q = H$, where H is a (left-open) half space.