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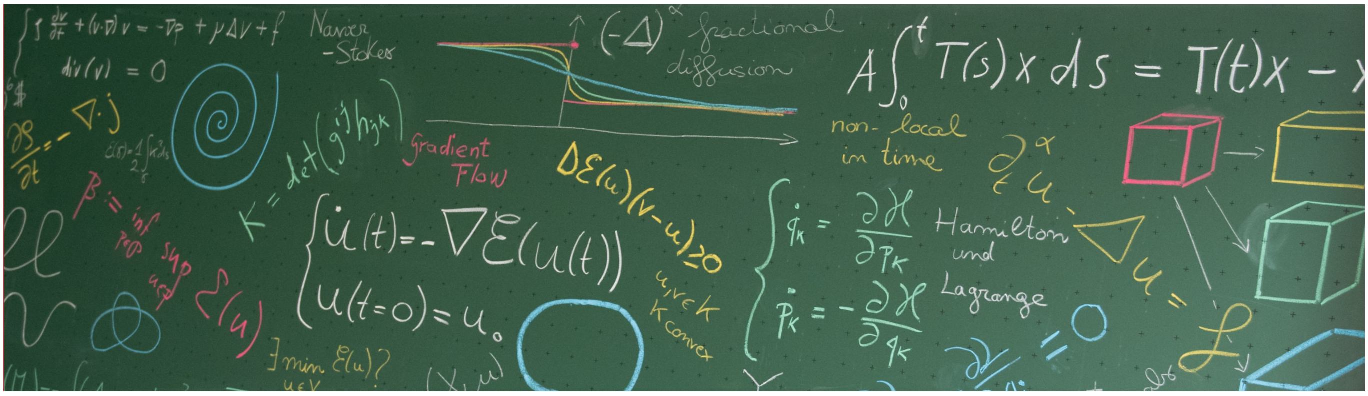
Gradient Flows and Variational Methods in PDEs

Ulm University, November 25–29, 2019



Ulm University, 25–29 November 2019 (as at 2019-11-27)

Timetable: *Gradient Flows and Variational Methods in PDEs* Winter School

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-9:00	**Registration** at H12 open from 8:00 **Opening** at 8:45				
9:00–10:00	**Brenier**	**Brenier**	**Chill**	**Bögelein**	**Chill**
10:00–11:00	**Chill**	**Mazzucato**	**Mazzucato**	**Mazzucato**	**Bögelein**
	Coffee break	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*
11:30–12:30	Wu/Hounkpe/Hopf	Knüttel/Kohout/Łasica	De Nitti/König/Kübler	Rupp/J. Weber/Santilli	Steenebrügge/Stephan **Closing**
12:30–14:00	*Lunch break*	*Lunch break*	*Lunch break*	*Lunch break*	
14:00–15:00	**Brenier**	Vorderobermeier/ Schätzler/Bárta	**Excursions** starting at 14:20 in city centre	**Bögelein**	
15:00–15:40	Dębiec/Abbatiello	Stollenwerk/Humpert		Sattig/F. Weber	
	Coffee break	*Coffee break*		*Coffee break*	
16:15–17:35	Cardona/Müller **Poster session** with pizza and beer starting at 17:00 in room H10	Rauchecker/Ghosh/Bonafini		Xu/Al Baba/ Correia Dos Santos/ Skrzeczkowski	
			Workshop dinner from 18:30 in Restaurant *Drei Kannen*		

Welcome to Ulm!

We cordially welcome you to Ulm University, a young and active research institution for science, technology and medicine. The Institute of Applied Analysis is pleased to host the international winter school "Gradient Flows and Variational Methods in PDEs". We hope you will enjoy the scientific programme, the interactions between young researchers and established experts, and of course the charming city of Ulm with its rich history and vibrant culture. The organizers and local staff will be happy to assist you with any questions or issues during your stay.

Registration

Every participant needs to register upon arriving at the workshop. Registration is open from **8 am on Monday morning** in front of H12. Later registration is possible on an individual basis, but please make sure you register as early as possible. At registration you need to choose whether you want to participate at an excursion and which of the two excursions you prefer. Moreover, you need to choose your meal for the workshop dinner. For more information on this matter you may look into the section "Social Events".

Transportation

Public Transport

You can easily reach the University by public transport from anywhere in Ulm. The closest bus stop for Ibis Hotel is “Theater Ulm” and the closest bus stop for B&B Hotel is “Ehinger Tor”. At either stop you can take the **tram number 2** with direction “Science Park II” and you have to leave the tram at the **station “Universität Süd”**. If you are planning to use the tram on 5 days for at least two times each day it is recommended to buy a weekly pass for €20.50 at one of the ticket vending machines. The tram number 2 also stops at the central station in between the aforementioned stations.

Taxi and Car

You can also reach the University by car. The only publicly available parking lot is the “Parkhaus Helmholtzstraße” located at Helmholtzstraße 5, 89075 Ulm. Using the parking spot for one day costs €5. Taxis are easily recognizable by the yellow “taxi” signs on top of their roof. You can order a taxi by calling ULM-TAXI +49 731 66066. A taxi from the central station to the university costs about €20.

At the University

Locations

Registration as well as all the lectures and talks will take place in lecture hall **H12** in cross section N24. The only exception is the poster session on Monday evening, which will take place in and around room **H10**. On the back cover you may find a schematic map that details the important locations: H12, H10, Bus/Tram stop "Universität Süd", and the Mensa and cafeterias.

Catering

During each of the coffee breaks we are going to provide coffee and some cookies. For lunch we are planning to go to the Mensa. The Mensa provides a variety of dishes and also vegetarian/vegan options. If you only want to have a small snack you can also go to one of the Cafeterias marked on the map. You need to pay for food in cash. As the next ATM is 500 m away we recommend to bring enough cash with you. We note that you need to use the checkout on the far left in the Mensa to pay for your food.

Internet

At most parts of the campus you are able to connect to the open WiFi "welcome". Note that this connection is unencrytped. If you have an eduroam profile installed on your device you should be able to use the WiFi "eduroam".

Social Events

Excursion on Wednesday afternoon

The hidden chambers in the choir towers of the Ulm Minster

In this guided tour we are going to take a closer look at the tallest church in the world. We are going to visit the main hall of the church and we will climb the choir towers afterwards, while learning more about the Ulm Minster and its history. **Prerequisites:** You need to bring good shoes, weatherproof clothing and you should be free from giddiness. We note that you need to climb some stairs during the tour and that there is no elevator. **Maximum Participants:** 40 (There are going to be two tours of 20 people each.) **Meeting Point:** We meet at 14:20 in front of the Ulm Minster. **Duration:** The tour lasts one and a half hours.

Guided tour through the Ulm Museum

In a guided tour we are going to visit parts of the Ulm Museum. In particular we are going to take a closer look at the famous lion man. The lion man is the oldest-known animal-shaped sculpture in the world, and the oldest-known uncontested example of figurative art. **Prerequisites:** None. **Maximum Participants:** 30. **Meeting Point:** We meet at 14:20 in front of the Ulm Museum. The address is Marktplatz 9, which can be found right next to the town hall of Ulm. **Duration:** The tour lasts one and a half hours.

Workshop Dinner

On Wednesday at 18:30 we are going to meet at the restaurant “Drei Kannen” for the workshop dinner. You can **choose between** either the **Schwabentafel** (a local dish containing a variety of food and in particular also some Käsespätzle) or **Käsespätzle** (vegetarian) for dinner. Please advise at registration if you have special dietary requirements so that we can make appropriate arrangements. The meals are paid by the organizers while you need to pay for your own drinks. The address is Hafenbad 31/1, 89073 Ulm. It is a 200 m walk from the nearest bus stop called “Justizgebäude” where the tram number 1 stops quite frequently. From the bus stop “Theater Ulm” it is a 800 m walk.

Monday, November 25

8:00–9:00	Registration at H12
9:00–10:00	Yann Brenier *Gradient Flows and Variational Methods for Fluid Motions*
10:00–11:00	Ralph Chill *Approximation and regularity for abstract gradient flow*
11:00–11:30	Coffee break
11:30–11:50	Jeremy Wu *Preliminary understandings of the Landau equation as a gradient flow*
11:50–12:10	Francis Hounkpe *A Parabolic Toy-Model for the Incompressible Navier-Stokes Equations*
12:10–12:30	Katharina Hopf *On the singularity formation and relaxation to equilibrium in 1D Fokker–Planck model with superlinear drift*
12:30–14:00	Lunch break
14:00–15:00	Yann Brenier *Gradient Flows and Variational Methods for Fluid Motions*
15:00–15:20	Tomasz Dębiec *Incompressible limit for a two-species tumour model*
15:20–15:40	Anna Abbatiello *Generalized solutions to models of viscous fluids*
15:40–16:15	Coffee break
16:15–16:35	Jorge Cardona *Measurable selection of semiflows*
16:35–16:55	Marius Müller *On Gradient Flows with Obstacles*
17:00	Poster session with pizza and beer in room H10

Tuesday, November 26

9:00–10:00	Yann Brenier *Gradient Flows and Variational Methods for Fluid Motions*
10:00–11:00	Anna Mazzucato *Optimal mixing in incompressible flows and irregular transport*
11:00–11:30	Coffee break
11:30–11:50	Sascha Knüttel *An alternative diffuse Approximation of the Wilmore-Functional*
11:50–12:10	James Kohout *The unique asymptotic limit problem for a gradient flow to minimal surfaces*
12:10–12:30	Michał Łasica *Existence of* 1*-harmonic map flow*
12:30–14:00	Lunch break
14:00–14:20	Nicole Vorderobermeier *On the analyticity of critical points for knot energies*
14:20–14:40	Leah Schätzler *Existence of variational solutions for doubly nonlinear equations of porous medium type*
14:40–15:00	Tomáš Bárta *Abstract wave equation with a general damping*
15:00–15:20	Kathrin Stollenwerk *Optimality conditions for the buckling of a clamped plate*
15:20–15:40	Ina Humpert *A Generalized Concept of Gradient Flows for PDEs with In- and Outflow of Mass*
15:40–16:15	Coffee break
16:15–16:35	Maximilian Rauchecker *The Mullins-Sekerka problem with contact angle*
16:35–16:55	Amrita Ghosh L^p*-Strong solution to fluid-rigid body interaction system with Navier slip boundary condition*
16:55–17:15	Mauro Bonafini *A variational scheme for hyperbolic obstacle problems*

Wednesday, November 27

9:00–10:00	Ralph Chill *Approximation and regularity for abstract gradient flow*
10:00–11:00	Anna Mazzucato *Optimal mixing in incompressible flows and irregular transport*
11:00–11:30	Coffee break
11:30–11:50	De Nitti *Differentiability properties of the flow associated with a nearly incompressible BV vector field*
11:50–12:10	Tobias König *Energy asymptotics in the Brézis-Nirenberg problem*
12:10–12:30	Joel Kübler *Symmetry breaking for spiraling solutions of nonlinear Schrödinger equations*
12:30–14:00	Lunch break
14:20	Excursions meeting point at Ulm Minster or at the museum Ulm
18:30	Workshop dinner from in the Restaurant “Drei Kannen”. Address: Hafenbad 31/1, 89073 Ulm

Thursday, November 28

9:00–10:00	Verena Bögelein *A Variational Approach for Evolution Problems*
10:00–11:00	Anna Mazzucato *Optimal mixing in incompressible flows and irregular transport*
11:00–11:30	Coffee break
11:30–11:50	Fabian Rupp *The constrained Łojasiewicz–Simon gradient inequality*
11:50–12:10	Jörg Weber *Optimal Control of the Relativistic Vlasov-Maxwell System*
12:10–12:30	Mario Santilli *Uniqueness of critical points of the anisotropic isoperimetric problem for finite perimeter sets*
12:30–14:00	Lunch break
14:00–15:00	Verena Bögelein *A Variational Approach for Evolution Problems*
15:00–15:20	Gabriel Sattig *Flexibility of Incompressible Transport*
15:20–15:40	Frederic Weber *The Fractional Laplacian has Infinite Dimension*
15:40–16:15	Coffee break
16:15–16:35	Qiang Xu *Layer potential methods in periodic homogenization theory*
16:35–16:55	Hind Al Baba *Fractional Powers of the Stokes operator with boundary conditions involving the pressure*
16:55–17:15	Matheus Correia dos Santos *Minimizing movement for a fractional porous medium equation in a periodic setting*
17:15–17:35	Jakub Skrzeczkowski *Parabolic PDEs in Musielak - Orlicz spaces: growth controlled by a function discontinuous in time.*

Friday, November 29

9:00–10:00	Ralph Chill *Approximation and regularity for abstract gradient flow*
10:00–11:00	Verena Bögelein *A Variational Approach for Evolution Problems*
11:00–11:30	Coffee break
11:30–11:50	Daniel Steenebrügge *Long-Time Existence of a Parametrization-Preserving Gradient Flow for Generalized Integral Menger Curvature in the Hilbert Case*
11:50–12:10	Artur Stephan *EDP-convergence for linear reaction-diffusion systems with different time scales*
12:10–12:25	Closing

Abstracts

Main Speakers

A Variational Approach for Evolution Problems

Verena Bögelein
University of Salzburg

In this minicourse we present a variational approach which applies to a huge variety of evolution problems. We start by introducing the notion of variational solution and explain the connection with weak solutions. Then we present different techniques to prove existence of variational solution in the simplest setting. These methods are subsequently used to treat more difficult parabolic equations and systems. In particular, we present a variational approach to the existence of solutions to equations of Porous Medium type, doubly nonlinear parabolic equations and parabolic systems in noncylindrical domains.

Gradient Flows and Variational Methods for Fluid Motions

Yann Brenier
CNRS, DMA Ecole Normale Superieure Paris

Lecture 1: The heat equation can be interpreted as the gradient flow of the Boltzmann entropy with respect to the metric of optimal transportation for volume forms, as well known since the work of Otto and collaborators. It can also be viewed as a degeneration of the Euler equations of isothermal flows through a suitable quadratic change of time. It is possible to derive more sophisticated parabolic equations either by using different functionals (for instance the so-called quantum drift diffusion equation is obtained by substituting the Fisher information for the Boltzmann entropy) or by using suitable generalizations of the standard optimal transport theory, as shown in the next 2 lectures.

Lecture 2: The Muskat equation (aka the incompressible porous medium equation) describes the motion of an inhomogeneous incompressible fluid in a porous medium and is a challenging model for convex integration theory. Then the gradient flow structure can be interpreted in terms of optimal *incompressible* transport and is based on the Helmholtz decomposition either in its standard linear form for vector fields or in its nonlinear form (polar factorization of maps).

Lecture 3: the Moffat model of magnetic relaxation for incompressible fluids has been introduced with the purpose of finding stationary solutions of the Euler equations while prescribing the topology of their streamlines, according to a program designed by Arnold and Moffatt in the 80s. It can also be interpreted as a gradient flow. There, the optimal transport setting is related to the concept of 1-currents instead of volume forms as in standard optimal transport theory.

In both cases, reduced models can be discussed in terms either of linear algebra or in terms of combinatorial optimization (linear and quadratic assignment problems) in close connection with the concept of Brockett flow.

Approximation and regularity for abstract gradient flow

Ralph Chill
TU Dresden

Many parabolic equations such as diffusion equations or Cahn-Hilliard equations related to phase separation processes can be written as abstract gradient systems of the form $\dot{u} + \partial\mathcal{E}(u) \ni 0$, where $\mathcal{E}$ is a semiconvex, lower semicontinuous energy function on a Hilbert space, taking values in the extended real line $\mathbb{R} \cup \{+\infty\}$. In this basic lecture we review the question of wellposedness of such gradient systems, that is, the generation of a gradient flow, including maximal regularity. We then consider the problem of continuous dependence of solutions on the energy function. We describe applications such as nonlinear diffusion equations, but also some more advanced recent models, their homogenisation, numerical analysis and diffusion equations on varying domains.

Optimal mixing in incompressible flows and irregular transport

Anna Mazzucato
Pennsylvania State University

I will discuss recent results on optimal mixing of passive scalars by incompressible flows. I will introduce differentiations measures of mixing and discuss known rates of mixing and examples of optimal mixers. These results are connected with recent advances in understanding transport by irregular vector fields. In particular, I will discuss an example of complete, instantaneous loss of regularity for solutions to linear transport equations with Sobolev advecting velocity.

Short Communications

Generalized solutions to models of viscous fluids

Anna Abbatiello[1], Eduard Feireisl[1, 2] and Antonín Novotný[3]
Technische Universität Berlin[1]; Academy of Sciences of the Czech Republic[2];
Université de Toulon[3]

We propose a new approach to models of general compressible viscous fluids based on the concept of dissipative solutions. These are weak solutions satisfying the underlying equations modulo a defect measure. A dissipative solution coincides with the strong solution as long as the latter exists (weak–strong uniqueness) and they solve the problem in the classical sense as soon as they are smooth (compatibility). We consider general models of compressible viscous fluids with non–linear viscosity tensor and non–homogeneous boundary conditions, for which the existence of global–in–time weak solutions is not known.

[1] A. Abbatiello, E. Feireisl, A. Novotný. Generalized solutions to models of compressible viscous fluids. *Forthcoming*.

Fractional Powers of the Stokes operator with boundary conditions involving the pressure

Hind Al Baba
Lebanese University Tripoli

Stokes and Navier-Stokes problems have been often studied with Dirichlet boundary condition. Nevertheless, in the opinion of engineers and physicists such a condition is not always realistic in industrial and applied problems of origin. Thus arises naturally the need to carry out a mathematical analysis of these systems with different boundary conditions, which best represent the underlying fluid dynamic phenomenology. Based on the study of the complex and fractional powers of the Stokes operator with pressure boundary condition, we carry out a systematic treatment of the Stokes problem with the corresponding boundary conditions in L^p-spaces.

Abstract wave equation with a general damping

Tomáš Bárta[1] and Eva Fašangová[2]
Charles University, Prague[1]; TU Dresden[2]

We study the equation

$$\ddot{u} + g(\dot{u}) + E'(u) = 0, \tag{1}$$

where $E \in C^1(V)$ is a potential and $g : V \to V'$ satisfying $\langle g(v), v\rangle \geq 0$ is a damping, $V \hookrightarrow H \hookrightarrow V'$ are Hilbert spaces. A key assumption is that E satisfies the Łojasiewicz–Simon gradient inequality

$$|E(u) - E(\varphi)|^{1-\theta} \leq C\|E'(u)\|_{V'}$$

for u in a neighborhood of an equilibrium φ with appropriate constants $C > 0$, $\theta \in (0, \frac{1}{2}]$. If $E(u) = \int_\Omega \|\nabla u(x)\|^2 + F(u(x), x)\, dx$, then (1) becomes a nonlinear wave equation

$$u_{tt} + g(u_t) - \Delta u - f(u, x) = 0$$

with $f(u, x) = \frac{\partial}{\partial u} F(u, x)$, whereas (1) becomes an ODE if $V = R^n$.

It is well-known that the Łojasiewicz–Simon inequality implies that any relatively compact solution to the gradient system $\dot{u} + E'(u) = 0$ converges to an equilibrium φ with the decay

$$\|u(t) - \varphi\| \leq \begin{cases} Ce^{-ct} & \text{if } \theta = \frac{1}{2}, \\ C(t+1)^{-\frac{\theta}{1-2\theta}} & \text{i}f\ \theta < \frac{1}{2}. \end{cases}$$

The same statement is valid for (1) if g is linear (see [1]), resp. if $\langle g(v), v\rangle \geq c\|v\|^2$. We focus on the case where g is very small so the last inequality is violated. We show that solutions still converge to φ if g is not too small, however the decay is slower and depends on g, see [2], [3].

[1] R. Chill, A. Haraux, and M. A. Jendoubi, *Applications of the Łojasiewicz-Simon gradient inequality to gradient-like evolution equations*, Anal. Appl. **7** (2009), 351–372.

[2] T. Bárta, E. Fašangová, *Convergence to equilibrium for solutions of an abstract wave equation with general damping function*, J. Diff. Equ. 260 (2016), no. 3, 2259 – 2274.

[3] T. Bárta, *Decay estimates for solutions of an abstract wave equation with general damping function*, Electron. J. Differential Equations, Vol. 2016 (2016), No. 334, pp. 1 – 17.

A variational scheme for hyperbolic obstacle problems

Mauro Bonafini[1], Van Phu Cuong Le[2], Matteo Novaga[3], Giandomenico Orlandi[4]
TUM[1]; Università di Trento[2]; Università di Pisa[3]; Università di Verona[4]

For an open bounded regular domain $\Omega \subset \mathbb{R}^d$, $d > 1$, and for $T > 0$, we consider the obstacle problem for the semilinear wave equation

$$\begin{cases} u_{tt} + (-\Delta)^s u + W'(u) = 0 & \text{in } (0,T) \times \Omega \\ u(t,x) = 0 & \text{in } [0,T] \times (\mathbb{R}^d \setminus \Omega) \\ u(0,x) = u_0(x) & \text{in } \Omega \\ u_t(0,x) = v_0(x) & \text{in } \Omega \end{cases} \tag{1}$$

where $(-\Delta)^s$ is the fractional Laplacian of order s, $s > 0$, W is a smooth non-negative potential with bounded second derivative and $u_0 \in \tilde{H}^s(\Omega)$, $v_0 \in L^2(\Omega)$ are suitable initial conditions. Given a one-sided obstacle $g\colon \Omega \to \mathbb{R}$, we use a semi-discrete approximation scheme to prove existence of a suitably defined weak solution to the obstacle problem associated to (1). The scheme allows to perform numerical simulations which give quite precise evidence of dynamical effects.

[1] M. Bonafini, M. Novaga and G. Orlandi. A variational scheme for hyperbolic obstacle problems, *Nonlinear Analysis* **188** (2019), pp. 389–404.

[2] M. Bonafini, VPC. Le, M. Novaga and G. Orlandi. On the obstacle problem for fractional semilinear wave equations, *preprint* (2019).

Measurable selection of semiflows

Jorge Cardona[1] and Lev Kapitanski[2]
TU Darmstadt[1]; University of Miami[2]

Consider a time independent dynamical system

$$\dot{u}(t) = F(u(t))\,. \tag{1}$$

Assume that for every initial condition a the solution of (1) with $u(0) = a$ is unique, then the semigroup property of the flow is naturally obtained, i.e.

$$u(t+s,a) = u(t,u(s,a))$$

In the cases when uniqueness is not given or is not yet proven, a set-valued map $S(a)$ with all the possible solutions with the same initial condition a can be defined. We proved an abstract result on the measurable selection of a flow with the semigroup property, our proof is motivated by the results of N.V. Krylov [1], and D. Stroock and S.R.S. Varadhan [2].

Some generalization will presented as well as applications to the incompressible Navier-Stokes system and other PDEs.

[1] N.V. Krylov, On the selection of a Markov process from a system of processes and the construction of quasi-difussion processes, *Izv. Akad. Nauk SSSR Ser. Mat.* **37** (1973) No. 3, pp. 691–708.

[2] D.W. Stroock and S.R.S. Varadhan, *Multidimensional Diffussion Processes*, Springer Verlag, Berlin, 1979.

Incompressible limit for a two-species tumour model

Tomasz Dębiec[1] and Markus Schmidtchen[2]

University of Warsaw[1]; Laboratoire Jacques-Louis Lions[2]

In recent years there has been an increasing interest in multi-phase models applied to tumour growth, which was traditionally modelled using a single equation describing the evolution of the abnormal cell density. We present a two-species model with coupling through the so-called Brinkman law, which is typically used in the context of visco-elastic media, where the velocity field is linked to the total population pressure via an elliptic equation. The same model for only one species has been studied in the past by Perthame and Vauchelet. We establish existence of solutions to the problem and the incompressible limit as the stiffness of the pressure law tends to infinity. Our approach, in one spatial dimension, relies on uniform BV-estimates.

Differentiability properties of the flow associated with a nearly incompressible BV vector field

Nicola De Nitti

Università degli Studi di Bari Aldo Moro (Italy)

The classical Cauchy-Lipschitz theorem guarantees existence, uniqueness, and Lipschitz regularity of the flow associated with a Lipschitz continuous vector field. In the last few decades – in view of several applications to fluid dynamics and to the theory of conservation laws – much effort has been put into the study of ODEs driven by vector fields which are not necessarily Lipschitz continuous, but belong only to some class of weak differentiability.

In this talk, which is based on a joint work with S. Bianchini (SISSA, Italy), we show that the flow associated with a nearly incompressible BV vector field is differentiable in measure.

L^p-Strong solution to fluid-rigid body interaction system with Navier slip boundary condition

Amrita Ghosh

Institute of Mathematics, Czech Academy of Science

I will discuss the existence of a strong solution of a coupled fluid and rigid body system and the corresponding L^p-theory. Precisely, I will consider a 3D viscous, incompressible non-Newtonian fluid, containing a 3D rigid body, coupled with (non-linear) slip boundary condition at the interface and show the well-posedness of this syetem.

[1] M. Geissert and K. Gotze and M. Hieber, L^p-theory for strong solutions to fluid-rigid body interaction in Newtonian and generalized Newtonian fluids, *Trans. Amer. Math. Soc.* **365 (3)**, 2013, pp. 1393–1439.

On the singularity formation and relaxation to equilibrium in 1D Fokker–Planck model with superlinear drift

José A. Carrillo[1], Katharina Hopf[2] and José L. Rodrigo[3]

Imperial College London[1]; WIAS Berlin[2]; University of Warwick[3]

We consider a class of Fokker–Planck equations with linear diffusion and superlinear drift enjoying a formal Wasserstein-like gradient flow structure with convex mobility function. In the drift-dominant regime, the equations have a finite critical mass above which the measure minimising the associated entropy functional displays a singular component. Our approach, which addresses the one-dimensional case, is based on a reformulation of the problem in terms of the pseudo-inverse distribution function. Using maximum type principles and a Perron method, we obtain global-in-time existence, uniqueness and regularity for the problem in the new variables. The emphasis of this talk lies on the question of how the above framework allows us to deduce the long-time asymptotic behaviour as well as the formation of singularities in finite-time.

A Parabolic Toy-Model for the Incompressible Navier-Stokes Equations

Francis Hounkpe
University of Oxford

In the seminar, I will talk about the following parabolic toy-model for the incompressible Navier-Stokes equations (we focus here, only, on the 3D model)

$$\begin{cases} \partial_t u - \Delta u + u \cdot \nabla u + \frac{1}{2}(\nabla \cdot u)u & = 0 \\ u = (u_1, u_2, u_3). \end{cases}$$

This model satisfies the same energy inequality, same scaling symmetry as the incompressible Navier-Stokes system and is moreover super-critical in dimension 3. I am not the first one to study the regularity question for a toy-model to the Navier-Stokes equations. In fact, this question has been extensively examined (see for instance [1,2,3,4]). But our model, unlike the ones in the papers mentioned above, has a nonlinearity with an algebra similar to the one in the incompressible Navier-Stokes system; and moreover, we are able to recover for our model some fine properties such as e.g. Caffarelli-Kohn-Nirenberg and the backward uniqueness theorems. I will present here some partial regularity results that this model shares with the incompressible model and other results that occur only for our model.

[1] T. Tao , Finite Time Blowup for an Averaged Three-dimensional Navier-Stokes Equation, *J. Amer. Math. Soc.* **29** (2016), pp. 601–674.

[2] S. Montgomery-Smith, Finite Time Blow-up for a Navier-Stokes like Equation, in *Proc. Amer. Math. Soc.* **129** (2001) pp. 3025–3029.

[3] I. Gallagher and M. Paicu, Remarks on the Blow-up of Solutions to a Toy model for the Navier-Stokes Equations, *Proc. Amer. Math. Soc.* **137** (2009) pp. 2075–2083.

[4] D. Li and Ya. Sinai , Blow ups of Complex Solutions of the 3D Navier–Stokes System and Renormalization Group method, *J. Eur. Math. Soc.* **10** (2008), pp. 267–313.

A Generalized Concept of Gradient Flows for PDEs with In- and Outflow of Mass

Ina Humpert[1], Martin Burger[2] and Jan-Frederik Pietschmann[3]

Westfälische Wilhelms-Universität Münster[1];
Friedrich-Alexander-Universität Erlangen-Nürnberg[2]; TU Chemnitz[3]

Motivated by the Benamou-Brenier formulation [1] and our previous work [2] we adapt the concept of gradient flows to equations where the total mass is not conserved. This happens due to both in- and outflow boundary terms and spatially distributed reaction terms that prevent us to exploit the classical gradient flow structure. In particular, we use the direct method of calculus of variations to derive this concept. Secondly a distance functional is proposed that generalizes the classical Wasserstein norm which is inappropriate for probability measures with different mass. In the end, we present a Fokker-Planck type example that fulfills the new generalized concept of gradient flows.

[1] Jean-David Benamou and Yann Brenier. A computational fluid mechanics solution to the Monge-Kantorovich mass transfer problem. *Numerische Mathematik*, 84(3):375- 393, January 2000.

[2] M. Burger, I. Humpert and Jan-Frederik Pietschmann, On Fokker-Planck Equations with In- and Outflow of Mass, *arXiv:1812.07064 [math]*, December 2018. arXiv: 1812.07064.

A Gradientfree Approximation of the Willmore Energy

Nils Dabrock, Sascha Knüttel and Matthias Röger
Technische Universität Dortmund

I will talk about a new phase field approximation of the Willmore energy. I start from a diffuse perimeter approximation considered by Amstutz-van Goethem and motivate from this an approximation of the Willmore energy. I show a Γ-$\limsup$ estimate for the approximation and justify by a formal asymptotic expansion that a corresponding L^2-Gradient Flow converges to the Willmore Flow.

Energy asymptotics in the Brézis-Nirenberg problem

Rupert L. Frank[1,2], Tobias König[2] and Hynek Kovařík [3]

California Institute of Technology[1]; Ludwig-Maximilians-Universität München[2]; Università degli studi di Brescia[3]

For a bounded open set $\Omega \subset \mathbb{R}^3$ we consider the minimization problem

$$S(a+\epsilon V) = \inf_{0 \not\equiv u \in H_0^1(\Omega)} \frac{\int_\Omega (|\nabla u|^2 + (a+\epsilon V)|u|^2)\, dx}{(\int_\Omega u^6 \, dx)^{1/3}}$$

involving the critical Sobolev exponent. The function a is assumed to be critical in the sense of Hebey and Vaugon. Under certain assumptions on a and V we compute the asymptotics of $S(a+\epsilon V) - S$ as $\epsilon \to 0+$, where S is the Sobolev constant. (Almost) minimizers concentrate at a point in the zero set of the Robin function corresponding to a and we determine the location of the concentration point within that set as well as the concentration speed.

We will also discuss analogous energy asymptotics for space dimension $N \geq 4$.

The unique asymptotic limit problem for a gradient flow to minimal surfaces

James Kohout

University of Oxford

The unique asymptotic limit problem asks whether a gradient flow that converges along a sequence of times $t_i \to \infty$ will converge to a *unique* asymptotic limit along every sequence of times $t \to \infty$. While examples of finite dimensional gradient flows that asymptote to a circle of critical points show that this property does not hold in general, Stanisław Łojasiewicz proved that this uniqueness of asymptotic limits holds for gradient flows of real analytic functions in finite dimensions. In his seminal work, Leon Simon provided a framework for tackling this problem for certain parabolic gradient flows of real analytic functionals. In the first part of the talk we will describe the role that the so-called Łojasiewicz-Simon inequality plays in the study of uniqueness of asymptotic limits. We will then discuss the unique asymptotic limit problem for a geometric flow which is designed to evolve a map describing a closed surface in a given target manifold into a parametrization of a minimal surface. This talk is based on joint work with Melanie Rupflin and Peter M. Topping.

Symmetry breaking for spiraling solutions of nonlinear Schrödinger equations

Oscar Agudelo[1], Joel Kübler[2] and Tobias Weth[2]
University of West Bohemia[1]; Goethe University Frankfurt[2]

We consider solutions of a nonlinear Schrödinger equation which are invariant with respect to a spiraling motion. This leads to the problem

$$-\Delta u - \frac{1}{\lambda^2}[x_1\partial_2 - x_2\partial_1]^2 u + u = |u|^{p-2}u \quad \text{in } \mathbb{R}^2,$$

where $\lambda > 0$ and $p > 2$. Noting that positive solutions are necessarily radial and consequently produce trivial spirals, we use variational methods to construct sign-changing solutions and study their properties.

As a main result, we show that a symmetry breaking phenomenon occurs in the sense that least energy sign-changing solutions are radial for λ close to zero, but become nonradial as $\lambda \to \infty$.

Existence of 1-harmonic map flow

Lorenzo Giacomelli[1], Michał Łasica[2] and Salvador Moll[3]
Sapienza University of Rome[1]; Polish Academy of Sciences[2]; University of Valencia[3]

We consider the functional $TV_\Omega^\mathcal{N}$ of total variation of maps from a domain Ω in $\mathbb{R}^m$ into a complete Riemannian manifold $\mathcal{N}$ embedded in $\mathbb{R}^N$. We define it as the lower semicontinuous envelope in $L^2(\Omega, \mathcal{N})$ of the functional given for $u \in C^1(\Omega, \mathcal{N})$ by

$$\int_\Omega |\nabla u|.$$

We report on recent progress concerning the existence of steepest descent curves of $TV_\Omega^\mathcal{N}$. We note that in general $TV_\Omega^\mathcal{N}$ is not geodesically semiconvex, hence existence is not provided directly by standard methods of constructing gradient flows, such as the theory of Meyer-Ambrosio-Gigli-Savare. Thus, we construct the curves as solutions to a system of PDEs, obtained as limits of regular approximations.

For Lipschitz initial data, we prove local existence of unique solutions in convex domains. We discuss conditions under which solutions can be continued indefinitely.

In the case $m = 1$, we obtain suitably defined global solutions for any datum of bounded variation whose jumps are not too big.

On Gradient Flows with Obstacles

Marius Müller
Ulm University

Classically, a gradient flow moves into the direction of steepest energy descent. What happens if the flow is confined by obstacles? When hitting an obstacle, the gradient direction might not be admissible.

The direction of steepest energy descent can now be found in two ways: Firstly, one can regard the flow as a gradient flow in a metric space, as commonly done in optimal transport theory. Secondly, one can view the problem as a parabolic obstacle problem and use a free boundary formulation with variational inequalities.

We will examine consistency of both approaches and identify a framework that allows several desirable structural results, such as slope control. Real world applications will only be outlined marginally, but form the centerpiece of [1].

[1] M. Müller, On Gradient Flows with Obstacles and Euler's Elastica. *Nonlinear Anal.* **192** (2020).

The Mullins-Sekerka problem with contact angle

Helmut Abels[1], Maximilian Rauchecker[1] and Mathias Wilke[2]
Universität Regensburg[1]; Universität Halle[2]

The Mullins-Sekerka problem for closed interfaces is widely studied since it appears naturally as a gradient flow of the area functional, as a sharp interface limit of the Cahn-Hilliard equation, and in physical models of phase changes. In this talk I will address the Mullins-Sekerka problem for interfaces with a ninety degree contact angle. In particular, I will show existence and uniqueness of strong solutions and discuss stability properties. This is joint work with Helmut Abels, Harald Garcke, and Mathias Wilke.

[1] H. Abels, M. Rauchecker, M. Wilke, Well-Posedness and qualitative behaviour of the Mullins-Sekerka problem with ninety-degree angle boundary contact,
arXiv 2019, arXiv:1902.03611.

[2] H. Garcke, M. Rauchecker, Stability analysis for stationary solutions of the Mullins-Sekerka flow with boundary contact, arXiv 2019, arXiv:1907.00833.

The constrained Łojasiewicz–Simon gradient inequality

Fabian Rupp
Ulm University

Since the pioneering work of L. Simon, the Lojasiewicz-Simon gradient inequality has been a powerful tool for studying asymptotic properties of gradient flows. Even though its most famous application is in the context of PDEs, the question whether a given energy on a Banach space does satisfy a Lojasiewicz-Simon gradient inequality can be answered from a purely functional analytic perspective, c.f. [1].

In many scientific models, it is natural to require that certain quantities remain conserved during the evolution process. This imposes some constraints on the model, e.g. the conservation of total mass. In order to apply the gradient inequality to the associated "constrained" gradient flow, one needs to prove a suitable version on a submanifold, modelling the constraint.

In our talk, we present sufficient conditions for the Lojasiewicz-Simon gradient inequality to hold on a submanifold of a Banach space and discuss the optimality of our assumptions. Our result provides a tool to study asymptotic properties of gradient flows with constraints.

[1] R. Chill, On the Łojasiewicz–Simon gradient inequality, *J. Funct. Anal.* **201** (2003), no. 2, pp. 572 – 601.

[2] F. Rupp, On the Łojasiewicz–Simon gradient inequality on submanifolds, *arXiv:1907.09292 [math.FA]* (2019).

Uniqueness of critical points of the anisotropic isoperimetric problem for finite perimeter sets

Mario Santilli
Augsburg Universität

Given a continuous positive integrand F on $\mathbf{R}^{n+1}$, the classical anisotropic isoperimetric problem is the minimization of the anisotropic perimeter functional

$$\mathcal{P}_F(E) = \int_{\partial^* E} F(\mathbf{n}(E,x))\, d\mathcal{H}^n x$$

among all sets of finite perimeter E in $\mathbf{R}^{n+1}$ with prescribed volume (here $\partial^* E$ is the reduced boundary of E and $\mathbf{n}(E,x)$ is the measure-theoretic exterior normal to E at x). It is well known that the global minimum is uniquely characterized up to translation by the Wulff shape associated to F, as proved by Jean Taylor in the 70's.

Instead of minima, a more subtle question asks to characterize the *critical points* of the anisotropic perimeter with prescribed volume. In this talk we present a uniqueness

result for these critical points: if F is an elliptic integrand of class $\mathcal{C}^3$ and $E \subseteq \mathbf{R}^{n+1}$ is a volume-constrained critical point of $\mathcal{P}_F$ such that

$$\mathcal{H}^n(\overline{\partial^* E} \setminus \partial^* E) = 0,$$

then E is equivalent to a finite union of disjoint (but possibly mutually tangent) open Wulff shapes.

This is a joint work with Antonio De Rosa and Sławomir Kolasiński.

Minimizing movement for a fractional porous medium equation in a periodic setting

Matheus C. Santos[1], Lucas C. F. Ferreira[2] and Julio V.-Guevara[3]

UFRGS[1]; Unicamp [2]; UCSP[3]

We consider a fractional porous medium equation

$$\begin{cases} \partial_t \rho + (-\Delta)^\sigma \rho^m = 0, & (x,t) \in \mathbb{T}^d \times (0,\infty) \\ \rho(0,x) = \rho_0(x), & x \in \mathbb{T}^d \end{cases}.$$

where $d \geq 1, 0 < \sigma < 1$, $0 < m \leqslant 2$ and $\mathbb{T}^d$ is the d-dimensional torus. The flow is studied in the space of periodic probability measures endowed with a non-local transportation distance constructed in the spirit of the Benamou-Brenier formula [1] and adapts the ideas from [2] and [3] for this case. For initial periodic probability measures, we show the existence of absolutely continuous curves that are generalized minimizing movements associated to Rényi entropy.

[1] J.-D. Benamou, Y. Brenier. *A computational fluid mechanics solution to the Monge-Kantorovich mass transfer problem*, Numer. Math. **84** (2000), no. 3, 375–393.

[2] M. Erbar, Gradient flows of the entropy for jump processes. *Ann. Inst. Henri Poincaré Probab. Stat.* **50** (2014), no. 3, 920–945.

[3] L. Roncal, P. Stinga. *Fractional Laplacian on the torus*. Commun. Contemp. Math. **18** (2016), no. 3, 1550033, 26 pp.

Flexibility of Incompressible Transport

Gabriel Sattig
Universität Leipzig

By the convex integration method we construct infinitely many solutions to the transport equation with incompressible flow field

$$\begin{aligned} \partial_t \rho + v \cdot \nabla \rho &= 0 \\ \nabla \cdot v &= 0 \end{aligned}$$

on $[0, T] \times \mathbb{T}^d$ (i.e. the flat torus) for any dimension $d \geq 2$. These solutions are neither Lagrangian nor renormalized (thus proving non-uniqueness) and can be chosen in the regularity class

$$\rho \in C_t^0 L_x^p,\ v \in C_t^0 W_x^{1,q} \text{ where } \frac{1}{p} + \frac{1}{q} > 1 + \frac{1}{d},$$

therefore contrasting the well-posedness result of DiPerna and P.-L. Lions. Moreover, if Besov conditions are considered the setting becomes similar to well-posedness statements based on the commutator estimate of Constantin, E and Titi.

In any case, the dichotomy is not sharp. We try to illuminate the “regularity gap” and give some implications for the wild behavior of low regularity Lagrangian flows, based on our result in comparison with a recent conditional uniqueness result by Caravenna and Crippa. Joint work with S. Modena and L. Székelyhidi.

Existence of variational solutions for doubly nonlinear equations of porous medium type

Leah Schätzler
Friedrich-Alexander-Universität Erlangen-Nürnberg

In this talk we discuss the existence of variational solutions to the Cauchy-Dirichlet problem with time dependent boundary values associated with doubly nonlinear parabolic equations of the type

$$\partial_t |u|^{m-1} u - \operatorname{div}(|\nabla u|^{p-2} \nabla u) = 0$$

in a bounded space-time cylinder $\Omega_T := \Omega \times [0, T] \subset \mathbb{R}^{n+1}$, $n \in \mathbb{N}$, with parameters $m \in (0, \infty)$ and $p \in (1, \infty)$. Note that this equation contains the parabolic p-Laplacian ($m = 1$) and the porous medium equation ($p = 2$) as special cases. The existence result relies on a nonlinear variant of the method of minimizing movements.

Parabolic PDEs in Musielak - Orlicz spaces: growth controlled by a function discontinuous in time.

Jakub Skrzeczkowski[1], Miroslav Bulíček [2] and Piotr Gwiazda [3]
University of Warsaw [1]; Charles University in Prague [2];
Institute of Mathematics, Polish Academy of Sciences[3]

We consider a general parabolic PDE with Dirichlet boundary condition and monotone operator A

$$u_t(t,x) = \mathrm{div} A(t,x,\nabla u(t,x)) + f(t,x) \text{ in } (0,T)\times\Omega,$$

where growth and coercivity is controlled by an N-function $M(t,x,\xi)$ depending on time:

$$M(t,x,\xi) + M^*(t,x,A(t,x,\xi)) \le c\, A(t,x,\xi)\cdot\xi,$$

where M^* is the convex conjugate of M. Note that for $M(t,x,\xi) = |\xi|^p$, we recover the classical L^p setting.

We do not make any assumption on the continuity in time for the N-function M. Using additional regularization effect coming from the equation, as in the case of renormalized solutions to the transport equation, we establish existence of weak solutions and in the particular case of isotropic N-function of the form $M(t,x,|\xi|)$, we also prove their uniqueness.

This general result applies to the equations studied in the literature like $p(t,x)$-Laplacian with p being discontinuous in time t.

Long-Time Existence of a Parametrization-Preserving Gradient Flow for Generalized Integral Menger Curvature in the Hilbert Case

Jan Knappman, Henrik Schumacher, Daniel Steenebrügge and Heiko von der Mosel
RWTH Aachen University

Generalized Integral Menger Curvature, introduced in [1], is a knot energy, i.e. a functional on the set of closed, embedded curves that punishes self-intersections. For certain parameters, the fractional Sobolev space associated with the energy is a Hilbert space, enabling us to define a gradient flow which evolves the original curve into a “nicer” shape.

With the method of *projected gradients*, see [2], and the *logarithmic strain* constraint introduced in [3], we obtain a lower bound T on the existence time of a modified gradient flow starting at a curve γ_0. As long as all initial curves η fulfil $|\eta'(x)| = |\gamma_0'(x)|$ for all x, we have that T is nonincreasing in the energy of η. The former condition is ensured by our modification to the flow, yielding long time existence via a continuation argument.

You can find the current version of the slides to the talk under https://www.instmath.rwth-aachen.de/~steenebruegge/files/Talk_Ulm.pdf.

[1] Simon Blatt and Philipp Reiter, Towards a regularity theory for integral Menger curvature. *Ann. Acad. Sci. Fenn. Math.*, 40(1):149–181, 2015.

[2] J. W. Neuberger, *Sobolev gradients and differential equations*, volume 1670 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 1997.

[3] Sebastian Scholtes, Henrik Schumacher, and Max Wardetzky, Variational Convergence of Discrete Elasticae. *arXiv e-prints*, page arXiv:1901.02228v1, Jan 2019.

Coarse-graining via EDP-convergence for linear reaction-diffusion systems with different time scales

Artur Stephan

WIAS Berlin

We study a linear reaction-diffusion system involving slow and fast reactions and investigate its behavior if some reaction rates tend to infinity. Assuming detailed balance, the problem can be understood as a gradient flow in Wasserstein space. We show how an effective limiting system can be rigorously derived and that the coarse-grained equation has again a gradient structure. The limit process is a reaction-diffusion system with mixed diffusion coefficients coarse-grained with respect to the microscopic equilibria of the fast reactions.

Optimality conditions for the buckling of a clamped plate

Kathrin Stollenwerk and Alfred Wagner

RWTH Aachen

For a domain $\Omega \subset \mathbb{R}^n$ we denote by

$$\Lambda(\Omega) := \inf \left\{ \frac{\int_\Omega |\Delta u|^2 dx}{\int_\Omega |\nabla u|^2 dx} : u \in H_0^{2,2}(\Omega) \right\}$$

the buckling load of Ω. In 1951, G. Polya and G. Szegö conjectured that among all domains of given volume, the ball minimizes the buckling load. This conjecture is still open.

We will prove the following uniqueness result: assume there exists a smooth domain that minimizes the buckling load among all domains of given volume and connected boundary. Then the domain must be a ball.

The proof uses the second domain variation and an inequality by L. E. Payne to establish the result.

On the analyticity of critical points for knot energies

Simon Blatt and Nicole Vorderobermeier
Universität Salzburg

How nice are critical knots of knot energies? We already know that critical points of the so-called Möbius energy with merely bounded energy are smooth. This leads to the question whether smooth critical points of the Möbius energy are also real analytic.

In this talk, we give a quick introduction to geometric knot theory with focus on the Möbius energy and present an essential technique, with which the open question on the analyticity of smooth critical points of the Möbius energy was solved [1].

To the best of the authors' knowledge, this is the first analyticity result in the context of non-local differential equations. The established approach can be helpful in studying the regularity of critical points of further knot energies, e.g. [2].

[1] S. Blatt and N. Vorderobermeier, On the analyticity of critical points of the Möbius energy, *Calculus of Variations and Partial Differential Equations* **58** (2019), no. 1, 58:16.

[2] N. Vorderobermeier, On the regularity of critical points for O'Hara's knot energies: From smoothness to analyticity, submitted to *Communications in Contemporary Mathematics*.

The fractional Laplacian has infinite dimension

Adrian Spener, Frederic Weber and Rico Zacher
Ulm University

We motivate and define the classical curvature-dimension inequality $CD(\kappa, N)$ of Bakry-Émery, where the constant $\kappa \in \mathbb{R}$ refers to the curvature and $N \in [1, \infty]$ to the dimension parameter. We say that an operator has finite dimension in the Bakry-Émery sense if $CD(\kappa, N)$ holds for some $N < \infty$ and say it has infinite dimension otherwise. An application of a finite dimension parameter are Li-Yau type inequalities, e.g. positive solutions to the heat equation on $\mathbb{R}^d$ satisfy a Li-Yau type inequality since the Laplacian on $\mathbb{R}^d$ satisfies $CD(0, d)$.

Does the fractional Laplacian satisfy $CD(0, N)$ with $N < \infty$? This question has recently been identified by Nicola Garofalo as a key open question towards the unsolved problem whether positive solutions to the fractional heat equation satisfy a Li-Yau type inequality.

In this talk we give a negative answer to Garofalo's question by showing that the fractional Laplacian has infinite dimension in the Bakry-Émery sense.

[1] A. Spener, F. Weber and R. Zacher, The fractional Laplacian has infinite dimension, *Comm. PDE* (2019), published online.

[2] A. Spener, F. Weber and R. Zacher, Curvature-dimension inequalities for non-local operators in the discrete setting, *Calc. Var. PDE* **58** (2019), no. 5, Paper No. 171.

Optimal Control of the Relativistic Vlasov-Maxwell System

Jörg Weber

Universität Bayreuth

The time evolution of a collisionless plasma is modeled by the relativistic Vlasov-Maxwell system, which couples the Vlasov equation (the transport equation) with the Maxwell equations of electrodynamics. The plasma particles are located in a bounded domain, for example a fusion reactor. In the exterior, there are external currents that may serve as a control of the plasma if adjusted suitably. Also, we allow material parameters, that is to say permittivity and permeability, which may depend on the space coordinate.

We discuss solution theory of the modeling nonlinear PDE system. Unfortunately, there is only a very weak solution concept and uniqueness of weak solutions to the initial value problem is not known.

This causes many problems when treating an optimal control problem such as following: On the one hand, particles hitting the boundary of their container and thus causing damage, and, on the other hand, exhaustive control costs are penalized. We prove existence of minimizers of the arising minimizing problem and give an approach to derive first order optimality conditions.

Preliminary understandings of the Landau equation as a gradient flow

José Carrillo[1], Matias Delgadino[2], and Jeremy Wu[3]

Imperial College London[1]; Pontifical Catholic University of Rio de Janeiro[2]; Imperial College London[3]

The Landau equation is a fundamental partial differential equation in kinetic theory. We focus on the spatially homogeneous setting where, for a parameter $\gamma \in [-4, 3]$, the Landau equation reads

$$\partial_t f = \nabla_v \cdot \left\{ f(v) \int_{\mathbb{R}^3} f(v_*)|v - v_*|^{\gamma+2}\Pi[v - v_*](\nabla_v \log f(v) - \nabla_{v_*} \log f(v_*))dv_* \right\}. \quad (1)$$

We aim to give a reinterpretation of (1) as a gradient flow inspired by a similar effort from Erbar for the closely related Boltzmann equation [1]. This allows for new numerical methods to be developed for (1) [2]. Another avenue of exploration is using the notion of geodesic convexity in gradient flows to hopefully answer the open uniqueness question which is not known in full generality for $\gamma < 0$ (although local and partial results are known [3]).

[1] M. Erbar, A gradient flow approach to the Boltzmann equation, *arXiv:1603.00540 [math.AP]* (2016).
[2] J. Carrillo, J. Hu, L. Wang, and J. Wu, A particle method for the homogeneous Landau equation, *arXiv:1910.03080 [math.AP]* (2019).
[3] N. Fournier, Uniqueness of Bounded Solutions for the Spatially Homogeneous Landau equation with a Coulomb potential, *Communications in Mathematical Physics* **299(3)**: (2010) pp. 765–782.

Layer potential methods in periodic homogenization theory

Qiang Xu
Max Planck Institute for Mathematics in the Sciences

This talk is devoted to study layer potential methods for elliptic operator with Neumann boundary conditions, arising in periodic homogenization theory. First, we will briefly introduce a periodic homogenization theory and related interesting problems. Then, we move to layer potential methods, in which we want to share an idea about how to approximate the singular kernel of trace operator in small-scales and large-scales. Finally, we will mention some applications. The content is mainly referred to [1] [2].

If you have further questions, please do not hesitate to contact us at

qiangxu@mis.mpg.de.

[1] C. Kenig and Z. Shen, Layer potential methods for elliptic homogenization problems, Comm. Pure Appl. Math. 64 (2011), no.1, 1-44.
[2] Q. Xu, P. Zhao and S. Zhou, The methods of layer potentials for general elliptic homogenization problems in Lipschitz domains, preprint, arXiv math:1801.09220v1.

Poster Session

- Ibrokhimbek Akramov (Ulm University)
  Energy Conservation for the Compressible Euler Equations with Vacuum

- Aras Bacho (Technical University of Berlin)
  Generalized Gradient Flows with Non-Variational Perturbation

- Sarah Biesenbach (RWTH Aachen University)
  Optimal L^1 relaxation to a bump for the Cahn-Hilliard equation on the line

- Nicola De Nitti (University of Bari)
  Sharp criteria for the waiting time phenomenon in solutions to the thin-film equation

- Xin Liu (Free University of Berlin)
  Global Strong Symmetric Solutions to Compressible Navier-Stokes Equations with Moving Boundary

- Marius Müller (Ulm University)
  Elasticae in the hyperbolic plane

- Nicola Picenni (Scuola Normale Superiore of Pisa)
  Mean curvature motion as a curve of maximal slope – The radial case

- Mikola Schlottke (Eindhoven University of Technology)
  From Gradient-Flow to Non-Gradient-Flow

- Jack Skipper (Ulm University)
  Lower Semi-Continuity for $\mathcal{A}$-Quasiconvex Functionals under Convex Restrictions

- William Vickery (University of Illinois at Chicago)
  Rough Paths: an Introduction

- Jeremy Wu (Imperial College London)
  Landau equation as a gradient flow thin-film equation

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