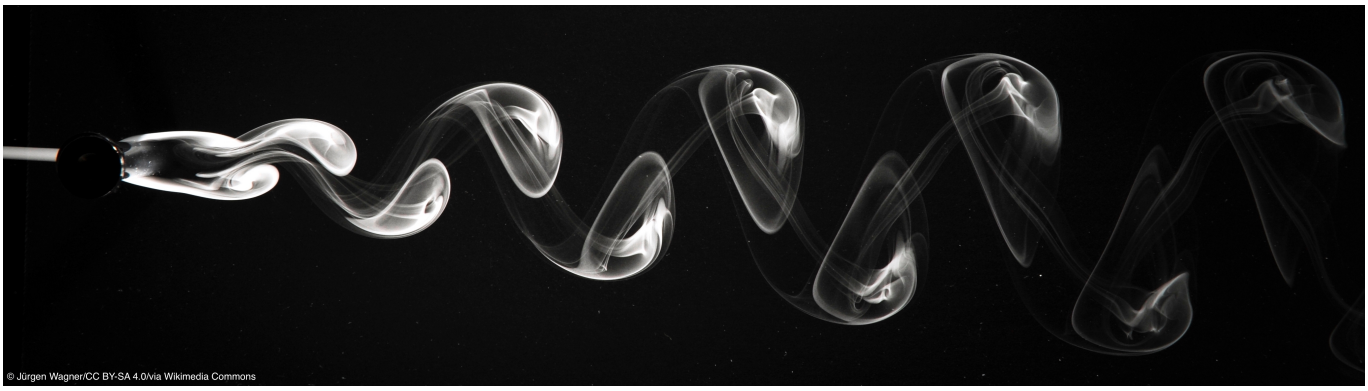

Horizons in non-linear PDEs

Ulm University, September 26 – 30, 2022

Ulm University, 26-30 September 2022 (as at 2022-09-23)

Timetable: *Horizons in non-linear PDEs* Summer School

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00-9:00	**Registration** in front of H22 open from 8:00				
9:00–10:00	**Mielke**	**Mielke**	**Rupflin**	**Carrillo**	**Fischer**
10:15–11:15	**Rupflin**	**Seis**	**Seis**	**Seis**	**Carrillo**
	Coffee break				
11:45–12:45	De Nitti/Schmitz Fernández-Jiménez/Bleitner	Müller/Luckas/Gallagher/Picenni	**Carrillo**	Zhao/Niebel/Ghosh/El Bahja	**Fischer**
					Closing
12:45–14:30	*Lunch break*				
14:30–15:30	**Mielke**	**Rupflin**		**Fischer**	
	Coffee break		**Excursions** starting at 15:30 in city centre	*Coffee break*	
16:00–17:00	Langer/De Anna/Gallenmüller Kampschulte/Nair/Schlierf	Esposito/Shah/Käse Barthwal/Huang/Kortum		Kouhkouh/Ménard/Wagner Biocic/Dembny/Heinze	
17:00–18:00	**Poster session** with pizza and beer starting at 17:30 in room H21				
			Workshop dinner from 18:30 in Restaurant *Drei Kannen*		

Welcome to Ulm!

We cordially welcome you to Ulm University, a young and active research institution for science, technology and medicine. The Institute of Applied Analysis is pleased to host the international summer school *Horizons in non-linear PDEs*. We hope you will enjoy the scientific program, the interactions between young researchers and established experts, and of course the charming city of Ulm with its rich history and vibrant culture. The organizers and local staff will be happy to assist you with any questions or issues during your stay.

Registration

Every participant needs to register upon arriving at the summer school. Registration is open from **8 am on Monday morning in front of H22** (see the map at the end of the booklet). Later registration is possible on an individual basis, but please make sure you register as early as possible. At registration you need to choose whether you want to participate in a guided city tour on Wednesday and which of the offered guided tours you prefer. Moreover, you have to decide if you attend the workshop dinner and choose which meal you prefer. For more information on this matter refer to the section *Social events*. If you have prepared a poster for the poster session, you can already hand it in during registration.
By the way, you will receive this brochure in printed form when you register, so you do not have to print it yourself.

Transportation

Public Transport

You can easily reach the university by public transport from anywhere in Ulm. The closest bus and tram stop for *Ibis Hotel* is *Theater Ulm* and the closest bus and tram stop for *B&B Hotel* is *Ehinger Tor*. At either stop you can take the tram line 2 headed to *Science Park II* and get off the tram at the station *Universität Süd*. If you are planning to use the tram on 5 days for at least two times each day, it is recommended to buy a weekly pass for €22.20 at one of the ticket vending machines at the stops or in the tram. The tram line 2 also stops at the central station in between the aforementioned stations. From the *Hotel am Rathaus* you can take bus line 5 in the direction of *Wissenschaftsstadt* or *Universität Süd*. Either you take a small detour and arrive approximately 30 minutes later directly at the stop *Universität Süd* or you change at the stop *Stadtwerke* to the tram line 2.

Taxi and Car

You can also reach the University by car. The only publicly available parking lot is the *Parkhaus Helmholtzstraße* located at Helmholtzstraße 5, 89075 Ulm. Using the parking spot for one day costs at least €5. Taxis are easily recognizable by the yellow *taxi* signs on top of their roof. You can order a taxi by calling ULM-TAXI +49 731 66066. A taxi from the central station to the university costs about €20.

At the University

Locations

Registration as well as all the lectures and talks will take place in lecture hall **H22**. The only exception is the poster session on Monday evening, which will take place in and around room **H21**. On the back cover you may find a schematic map with the important locations: H22, H21, bus/tram stop *Universität Süd*, the Mensa and cafeterias.

Catering

During each of the coffee breaks we are going to provide coffee, beverages and cookies. For lunch we are planning to go to the Mensa. The Mensa provides three different dishes among which are also vegetarian and vegan options. A menu is available at this link and is displayed including the information vegetarian/vegan/pork/beef in front of the Mensa. Unfortunately, we cannot cover the lunch expense. Please use the checkout on the far left in the Mensa to pay for your food. If you are a student, you need to bring an official-looking (stamped!) certificate of enrollment of your home institution in order to qualify for reduced student-prices at the Mensa.
If you only want to have a small snack, you can also go to the *Cafeteria Southside*. The Mensa and the Cafeteria Southside are marked on the map on the back of this booklet.
A third option is the food truck which is located at the tram and bus stop *Universität Süd*. Here you can find grilled sausage, curry sausage, soy-wheat Schnitzel, French fries and daily changing vegetarian or vegan offers.
Please note that everywhere at the university only card payment is possible.

Internet

At most parts of the campus you are able to connect to the open WiFi *welcome*. Note that this connection is unencrytped. If you have an eduroam profile installed on your device, you should be able to use the WiFi *eduroam*.

Social Events

Excursion on Wednesday afternoon

On Wednesday afternoon we offer participation in one of three city tours with different focuses. When registering on Monday, you choose one of the three city tours listed below. For all three city tours, the meeting point is at 15:30 in front of the Minster.

Classic city tour with a visit to the Minster

During an approximately 90-minute city tour with a guide, you will learn interesting facts throughout the history and present of the two-state town Ulm, visit the Minster (also inside), the historic town hall, the picture-perfect Fischerviertel and Gerberviertel and the so called Neue Mitte.

Guided tour through the historic district including Fischerviertel and Gerberviertel

This approximately 90-minute guided city tour through the Fischerviertel and the Gerberviertel is impressive. Winding streets, charming squares, romantic bridges over streams, the most crooked hotel in the world, you feel transported back to a bygone era...

Middle Ages meets modernity

Ulm is over 1150 years old and has a long, traditional history. But the city has remained young and shows its modern side in a new city-center with modern, impressive, relatively young buildings. This guided city tour takes you on a journey through time between the Middle Ages, Baroque, Renaissance and the present.

Workshop Dinner

On Wednesday at 18:30 we are going to meet at the restaurant *Drei Kannen* for the workshop dinner. If you prefer to eat vegetarian or vegan, you can indicate this during registration. Please advise at registration if you have special dietary requirements so that we can make appropriate arrangements. The meals are paid by the organizers while you need to pay for your own drinks. The address is Hafenbad 31/1, 89073 Ulm. It is a 200 m walk from the nearest bus and tram stop called *Justizgebäude* where the tram number 1 stops quite frequently. From the bus and tram stop *Theater Ulm*, where the tram number 2 stops, it is a 800 m walk.

Monday, September 26

8:00–9:00 Registration in front of H22

9:00–10:00 Alexander Mielke
Lecture 1: *Gradient flows: existence and Γ-convergence via the energy-dissipation principle*

10:15–11:15 Melanie Rupflin
Lecture 1: *Quantitative estimates for variational problems*

11:15–11:45 Coffee break

11:45–11:57 Nicola De Nitti
Nonlocal regularizations of conservation laws

12:00–12:12 Lina Sophie Schmitz
Analysis of a soap film catenoid driven by an electrostatic force

12:15–12:27 Alejandro Fernández-Jiménez
Concentration phenomena arising in Aggregation Fast-Diffusion equations

12:30–12:42 Fabian Bleitner
Navier Slip Boundary Conditions for the Rayleigh-Bénard Problem in Two Dimensions

12:45–14:30 Lunch break

14:30–15:30 Alexander Mielke
Lecture 2: *Gradient flows: existence and Γ-convergence via the energy-dissipation principle*

15:30–16:00 Coffee break

16:00–16:12 Leonie Langer
A dynamic approach to heterogeneous elastic wires

16:15–16:27 Francesco De Anna
On thermodynamic extensions of the Cahn-Hilliard equation

16:30–16:42 Dennis Gallenmüller
Diatomic measure-valued solutions of the isentropic Euler system: When do they come from a sequence of weak solutions?

16:45–16:57 Malte Kampschulte
Variational approaches to fluid-structure interaction

17:00–17:12 Gokul Gopan Nair
Energy Minimizing Configurations for Highly Deformable Elastic Surfaces

17:15–17:27 Manuel Schlierf
The Willmore flow with Dirichlet boundary data

17:30 Poster session with pizza and drinks in room H21

Tuesday, September 27

9:00–10:00 Alexander Mielke
Lecture 3: *Gradient flows: existence and Γ-convergence via the energy-dissipation principle*

10:15–11:15 Christian Seis
Lecture 1: *Regularity and mixing of passive scalars*

11:15–11:45 Coffee break

11:45–11:57 Marius Müller
Elliptic equations involving surface measures

12:00–12:12 Michelle Luckas
Global solutions for fluid-elastic interaction with small data

12:15–12:27 Ed Gallagher
The ∞-elastica problem on a Riemannian manifold

12:30–12:42 Nicola Picenni
Asymptotic behavior of minimizers for a singular perturbation of the one-dimensional Perona-Malik functional

12:45–14:30 Lunch break

14:30–15:30 Melanie Rupflin
Lecture 2: *Quantitative estimates for variational problems*

15:30–16:00 Coffee break

16:00–16:12 Francesco Esposito
Monotonicity of positive solutions to quasilinear elliptic equations in half-spaces with a changing-sign nonlinearity The current font size is: 12pt

16:15–16:27 Sarswati Shah
Weakly Compressible Two-layer Shallow-Water Flows

16:30–16:42 Philipp Käse
Surprising solutions to the ODE of constant mean curvature in homogeneous spaces

16:45–16:57 Rahul Barthwal
Existence and regularity of solutions of a supersonic-sonic patch arising in axisymmetric relativistic transonic flow

17:00–17:12 Hui Huang
A Mean-Field Optimal Control Approach to the Training of NeurODEs

17:15–17:27 Joshua Kortum
Concentration-cancellation for the simplified Ericksen-Leslie model

Wednesday, September 28

9:00–10:00 Melanie Rupflin
Lecture 3: *Quantitative estimates for variational problems*

10:15–11:15 Christian Seis
Lecture 2: *Regularity and mixing of passive scalars*

11:15–11:45 Coffee break

11:45–12:45 José A. Carrillo
Lecture 1: *Global minimizers of Interaction Energies*

12:45–14:30 Lunch break

15:30 Excursions starting at 15:30 in city center

18:30 Workshop dinner in the restaurant *Drei Kannen*
Address: Hafenbad 31/1, 89073 Ulm

Thursday, September 29

9:00–10:00 José A. Carrillo
Lecture 2: *Global minimizers of Interaction Energies*

10:15–11:15 Christian Seis
Lecture 3: *Regularity and mixing of passive scalars*

11:15–11:45 Coffee break

11:45–11:57 Xianliang Zhao
Non-exchangeable interacting diffusions with singular kernels

12:00–12:12 Lukas Niebel
Kinetic maximal L^p-regularity

12:15–12:27 Nibedita Ghosh
Global in time existence of the classical solutions for the system of multi-species transport with mass action kinetics

12:30–12:42 Hamid El Bahja
Regularity and existence of solutions of a parabolic double phase equation with variable growth

12:45–14:30 Lunch break

14:30–15:30 Julian Fischer
Lecture 1: *Interface evolution equations, phase-field models, and sharp-interface limits*

15:30–16:00 Coffee break

16:00–16:12 Hicham Kouhkouh
A viscous ergodic problem in noncompact setting: HJB equation and MFG

16:15–16:27 Raphael Wagner
Statistical solutions of the 2D incompressible Euler equations

16:30–16:42 Matthieu Ménard
Mean-field limit derivation of a gyrokinetic spray model

16:45–16:57 Ivan Biocic
Semilinear Dirichlet problem for subordinate spectral Laplacian

17:00–17:12 Mateusz Dembny
On optimal Harnack bounds for a fractional heat equation

17:15–17:27 Georg Heinze
Nonlocal Cross-Interaction Systems on Graphs: Nonquadratic Finslerian Structure and Nonlinear Mobilities

Friday, September 30

9:00–10:00 Julian Fischer
Lecture 2: *Interface evolution equations, phase-field models, and sharp-interface limits*

10:15–11:15 José A. Carrillo
Lecture 3: *Global minimizers of Interaction Energies*

11:15–11:45 Coffee break

11:45–12:45 Julian Fischer
Lecture 3: *Interface evolution equations, phase-field models, and sharp-interface limits*

12:45–13:00 Closing

Abstracts

Main Speakers

Global minimizers of Interaction Energies

José A. Carrillo
University of Oxford

I will review the existence and uniqueness of global minimizers for interaction energy functionals. Euler-Lagrange equations in the infinity wasserstein distance will be discussed. Based on linear convexity/concavity arguments, qualitative properties of the global minimizers will also be treated. Anisotropic singular potentials appearing in dislocations will be shown to have rich qualitative properties with loss of dimension and ranges of explicit minimizers. A large part of the course will be based on several works in collaboration with Ruiwen Shu (University of Oxford).

Interface evolution equations, phase-field models, and sharp-interface limits

Julian Fischer
Institute of Science and Technology Austria

PDEs that describe the evolution of an interface arise naturally in physics and geometry. Notable examples in physics include the description of the flow of two immiscible fluids (like oil and water) or models for the evolution of phase boundaries. A important example in geometry is mean curvature flow, the most natural gradient flow of the interface area functional.

Among the key challenges in the analysis and numerical approximation of interface evolution problems are topological changes, which occur naturally in many evolutions. A widespread approach to approximate interface evolution problems by a PDE posed on the entire space are phase-field models, like for instance the Allen-Cahn equation, circumventing the problem posed by topology changes. However, it is often a difficult task to prove that a given phase-field model in fact approximates the original interface evolution problem in the so-called sharp-interface limit, even prior to any topological change.

In this lecture series, we present a recent approach to sharp-interface limits that is based on the relative entropy method. The relative entropy controls the deviation of the phase-field model from the interface evolution problem in terms of an energy-like quantity. We illustrate important applications of this method, in particular a surprisingly short proof of convergence of the Allen-Cahn equation towards mean curvature flow prior to topology changes.

Gradient flows: existence and Γ-convergence via the energy-dissipation principle

Alexander Mielke

WIAS Berlin, Humboldt-Universität zu Berlin

In the first lecture we present basic examples and definitions of (generalized) gradient systems, gradient-flow equations and several weak formulations in Hilbert, Banach or metric spaces. A triple $(X, \mathcal{E}, \mathcal{R})$ is called a gradient system if X is a metric space, $\mathcal{E}$ is a lower semicontinuous energy functional, and $\mathcal{R}$ is a dissipation structure (cf. [AGS05,Mie13, San17]). In the main situation of a Banach space the gradient-flow equation is a given by $0 \in \partial_{\dot u}\mathcal{R}(u, \dot u) + \mathrm{D}\mathcal{E}(u)$. The point is that one PDE $\dot u = \mathcal{A}(u)$ can be written as a gradient-flow equation with respect to several gradient systems. Thus, the gradient system contains additional (physical) information that is not contained in the PDE.

The second lecture concerns a general existence theory for gradient flows which is based on the minimizing movement scheme. The approach from [RoS06,MRS13] allows for general dissipation potentials $\mathcal{R}(u, \cdot)$ with nontrivial dependence on the state u and general non-semiconvex energies $\mathcal{E}$. The theory relies on the Energy-Dissipation Principle (EDP), the De Giorgi (also called variational) interpolant, and the abstract chain rule. A potential application is viscoplasticity considered in [MRS18].

The final lecture concerns evolutionary Γ-convergence presenting certain generalizations and refinements of the Sandier-Serfaty approach [SaS04]. In particular, we introduce the notion of EDP-convergence, i.e. convergence of gradient systems in the sense of the energy-dissipation principle, see [Mie16, MMP21, FrL21]. We discuss an example where for a given PDE with a small parameter ε there are two different gradient structures, such that both EDP-converge for $\varepsilon \to 0$, but the two effective equations obtained in the limit are different.

[AGS05] L. Ambrosio, N. Gigli, and G. Savaré, *Gradient flows in metric spaces and in the space of probability measures*, Lectures in Mathematics ETH Zürich, Birkhäuser Verlag, Basel, 2005.

[FrL21] T. Frenzel and M. Liero: *Effective diffusion in thin structures via generalized gradient systems and EDP-convergence*. Discr. Cont. Dynam. Systems Ser. S **14**:1 (2021) 395–425.

[Mie13] A. Mielke: *Thermomechanical modeling of energy-reaction-diffusion systems, including bulk-interface interactions*. Discr. Cont. Dynam. Systems Ser. S **6**:2 (2013) 479–499.

[Mie16] ———, *On evolutionary Γ-convergence for gradient systems (Ch. 3)*, Macroscopic and Large Scale Phenomena: Coarse Graining, Mean Field Limits and Ergodicity (A. Muntean, J. Rademacher, and A. Zagaris, eds.), Springer, 2016, Proc. of Summer School in Twente University, June 2012, pp. 187–249.

[MMP21] A. Mielke, A. Montefusco, and M. A. Peletier: *Exploring families of energy-dissipation landscapes via tilting — three types of EDP convergence*. Contin. Mech. Thermodyn. **33** (2021) 611–637.

[MRS13] A. Mielke, R. Rossi, and G. Savaré: *Nonsmooth analysis of doubly nonlinear evolution equations*. Calc. Var. Part. Diff. Eqns. **46**:1-2 (2013) 253–310.

[MRS18] ———: *Global existence for viscoplasticity at finite strain*. Arch. Rat. Mech. Anal. **227** (2018) 423–475.

[RoS06] R. Rossi and G. Savaré: *Gradient flows of non convex functionals in Hilbert spaces and applications*. ESAIM Control Optim. Calc. Var. **12** (2006) 564–614.

[San17] F. Santambrogio: *{Euclidean, metric, Wasserstein} gradient flows: an overview*. Bull. Math. Sci. **7**:1 (2017) 87–154.

[SaS04] E. Sandier and S. Serfaty: *Gamma-convergence of gradient flows with applications to Ginzburg-Landau*. Comm. Pure Appl. Math. **LVII** (2004) 1627–1672.

Quantitative estimates for variational problems

Melanie Rupflin
University of Oxford

For geometric variational problems it is often important to not only analyse critical points, but to obtain a more general understanding of the energy landscape near points that either "almost minimise" a given energy or that are "almost critical points" of E. For the former it is natural to ask whether a map u whose energy $E(u)$ is close to the minimal possible energy E_{min} must necessarily be close to a minimiser of E and if so how we can obtain a quantitative version of this statement which allows us to bound the distance to the set of minimisers in terms of the energy defect $E(u) - E_{min}$.

On the other hand in the study of almost critical points, and the related asymptotic analysis of gradient flows, it is natural to ask whether smallness of $\nabla E(u)$ ensures closeness to the set of critical points. One is in particular interested in establishing so called Lojasiewicz inequalities (also called quantitative stability estimates), which bound the distance from the set of critical points as well as the energy defect by a power of the norm of $\nabla E(u)$. We will discuss how such one can prove Lojasiewicz estimates even in some singular settings, such as for almost harmonic maps that form simple singularities, and how one can combine gradient flow techniques and Lojasiewicz estimates to obtain quantitative rigidity estimates, e.g. for maps of a given degree between spheres whose energy is almost minimal.

Regularity and mixing of passive scalars

Christian Seis

University of Münster

In this minicourse, I will give an overview on recent results for linear transport equations with rough coefficients. For instance, we will discuss the optimal regularity that is propagated if the velocity is merely Sobolev regular: In this case, only derivatives of logarithmic order can be controlled. We will also be concerned with mixing phenomena in fluids. Here, optimal rates of mixing and enhanced dissipation can be derived.

Short Communications

Existence and regularity of solutions of a supersonic-sonic patch arising in axisymmetric relativistic transonic flow

Rahul Barthwal and T. Raja Sekhar
Indian Institute of Technology Kharagpur

We prove the existence and regularity of a smooth solution for a supersonic-sonic patch arising in modified Frankl problem in the study of three dimensional axisymmetric steady isentropic relativistic transonic flows over a symmetric airfoil. We consider a general convex equation of state which makes this problem complicated as well as interesting in the context of general theory for transonic flows. Such type of patches appears in many transonic flows over an airfoil and flow near the nozzle throat. The main difficulty in this article is the coupling of nonhomogeneous terms due to axisymmetry and the sonic degeneracy for the relativistic flow. However, using the well received characteristic decompositions of angle variables and a partial hodograph transformation, we prove the existence and regularity of solutions in partial hodograph plane first and then by using an inverse transformation, we construct a smooth solution in the physical plane and discuss the uniform regularity of solution up to the associated sonic curve. Finally, we also discuss the uniform regularity of the sonic curve.

Semilinear Dirichlet problem for subordinate spectral Laplacian

Ivan Biocic
University of Zagreb

We study semilinear problems in bounded $C^{1,1}$ domains for non-local operators with a boundary condition. The operators cover and extend the case of the spectral fractional Laplacian. We also study harmonic functions with respect to the non-local operator and boundary behaviour of Green and Poisson potentials.

Navier Slip Boundary Conditions for the Rayleigh-Bénard Problem in Two Dimensions

Fabian Bleitner
Universität Hamburg

Rayleigh-Benard convection describes the flow of a buoyancy driven fluid between a heated plate on the bottom and a cooled plate on top. For the velocity field often times no-slip or free-slip boundary conditions are used. In this talk we study the Navier-slip boundary conditions for the two dimensional Rayleigh-Bénard problem, which interpolate between the no- and free-slip case. The main focus is on regularity results and bounds on the ratio between convective and conductive heat transport, given by the Nusselt number, with respect to the Rayleigh number, describing the strength of the buoyancy force.

On thermodynamic extensions of the Cahn-Hilliard equation

Francesco De Anna
University of Würzburg

The Cahn-Hilliard equations is a fundamental model that describes the phase-separation processes of binary mixtures. In this talk we present a recent result, which concerns the extension of this theory to binary media, when the underlying temperature is not constant. We derive several models through a unified formalism, which is framed in terms of variational approaches and thermodynamic principles. These systems are all consistent with the main laws of thermodynamics. Secondly we determine some analytical results, showing that one of the derived models might be more desirable to the well-posedness theory of classical solutions, a property that might be natural as a selection criteria.

On optimal Harnack bounds for a fractional heat equation

Mateusz Dembny
University of Warsaw

Consider the linear heat equation $\partial_t u - \triangle u = 0$. The celebrated Li-Yau inequality states that for positive solutions we have $\triangle \log u \geq -\frac{n}{2t}$. By integrating this inequality along a straight space-time interval between two points, we may deduce the sharp Harnack estimate. In recent years there has been a considerable interest in extending the Li-Yau technique to the context of non-local diffusion, and in particular to the fractional heat equation $\partial_t u + (-\triangle)^\alpha u = 0$. Finding the fractional Li-Yau estimate still poses a challenge, but in the literature there are several Harnack inequalities concerning non-local diffusion. Our goal is to find an optimal Harnack inequality, in the sense that it yields an identity when applied to a fundamental solution. The presentation shows the progress made on this topic, and is based on the joint work with Mikołaj Sierżęga.

Nonlocal regularizations of conservation laws

Nicola De Nitti
FAU Erlangen-Nürnberg

We present some recent results on the problem of approximating a scalar conservation law by a conservation law with nonlocal flux. As convolution kernel in the nonlocal flux, we consider an exponential-type approximation of the Dirac distribution. We prove that the (unique) weak solution of the nonlocal problem converges strongly in $C(L^1_{loc})$ to the entropy solution of the local conservation law. This talk is based on joint works with G. M. Coclite, J.-M. Coron, A. Keimer, and L. Pflug.

Regularity and existence of solutions of a parabolic double phase equation with variable growth

Hamid El Bahja
AIMS South Africa

We study the Cauchy-Dirichlet problem for a class of nonlinear parabolic double phase equations with nonstandard growth conditions where the modulating coefficient is bounded away from zero. We prove theorems of existence and uniqueness of weak solutions in suitable Orlicz-Sobolev spaces, derive global and local in time L^∞ bounds for the weak solutions.

Monotonicity of positive solutions to quasilinear elliptic equations in half-spaces with a changing-sign nonlinearity

Francesco Esposito
University of Calabria

In this talk I will discuss about the monotonicity of positive solutions to quasilinear elliptic equations in half-spaces under zero Dirichlet boundary conditions, involving the p-Laplace operator for $\frac{2N+2}{N+2} < p < 2$ and for a general class of regular changing-sign nonlinearities f. The techniques used in the proof of the main result are based on a fine use of comparison and maximum principles and on an adaptation of the celebrated moving plane method to quasilinear elliptic equations in unbounded domains.

Concentration phenomena arising in Aggregation Fast-Diffusion equations

Alejandro Fernández-Jiménez
University of Oxford

Over the last two decades, a lot of attention has been given to the Aggregation-Diffusion Equation $\partial_t \rho = \operatorname{div}(\rho \nabla (U'(\rho) + V + W * \rho))$. This family of equations has become popular because, amongst others, it models the mean-field limit of systems with a large number of interacting particles arising in biology. Because of that, there has been a lot of interest concerning the study of the asymptotic behaviour as $t \to \infty$, discussing the existence or not of Dirac deltas, which are usually described as concentration phenomena. This talk will focus on Fast Diffusion ($U(\rho) = \rho^m/(m-1)$ with $0 < m < 1$) and V, W suitable regular. The Fast-Diffusion case presents a challenge because the free energy associated to this problem is no longer convex and gradient flow techniques do not guarantee convergence to the minimiser. Therefore, in order to overcome this difficulty we have to rely on compactness arguments as well. We will review some of the literature and we will show how to use these techniques to obtain the asymptotic behaviour. The talk presents ongoing joint work with Prof. J.A. Carrillo and Prof. D. Gómez-Castro.

The ∞-elastica problem on a Riemannian manifold

Ed Gallagher
University of Bath

In this talk, based on the preprint of the same name (`https://arxiv.org/abs/2202.07407`), we ask how to minimise the L^∞ (supremum) norm of the curvature of curves satisfying boundary conditions and a length constraint which live on a given Riemannian manifold. This builds on a paper of Moser considering the problem for Euclidean curves. Using the method of L^p approximation we show that the solutions of our problem must satisfy a given ODE system, derived as the limit of the L^p Euler-Lagrange equations as p tends to infinity. This system gives us some geometric information about our minimisers and in particular implies that their curvature takes on at most two values. The methods used are applicable in higher-dimensional problems too, leading to systems of PDEs rather than ODEs, but for simplicity and to most clearly illustrate the tools used we present here the one-dimensional case.

Diatomic measure-valued solutions of the isentropic Euler system: When do they come from a sequence of weak solutions?

Dennis Gallenmüller
Ulm University

Is every measure-valued solution of the isentropic Euler system generated by a sequence of weak solutions? In this short talk, we tackle this question for the example of measure-valued solutions consisting only of two Dirac-deltas. When we give sufficient and necessary conditions, the wave-cone connectedness of the underlying states turns out to be an important criterion which can be easily checked in the case of two space dimensions.

Global in time existence of the classical solutions for the system of multi-speciestransport with mass action kinetics

Nibedita Ghosh, Hari Shankar Mahato
IIT Kharagpur

The goal of this work is to establish the global existence of nonnegative clas-sical solutions in all dimensions for a system of highly nonlinear reaction-diffusion equations.We address the case for different diffusion coefficients and the system of reversible reaction-swith non-homogeneous Neumann boundary conditions. The systems are assumed to satisfyonly the mass control condition and to have locally Lipschitz nonlinearities with arbitrarygrowth. The key aspect of this work is that we didn't assume that the diffusion coefficients areclose to each other. We utilize the duality method and the regularization of the heat operatorto derive the result. We also illustrate the case where the diffusion coefficients are required tobe measurable and uniformly bounded. The application includes concrete corrosion in sewerpipes or sulfate corrosion in sewer pipes.

Nonlocal Cross-Interaction Systems on Graphs: Nonquadratic Finslerian Structure and Nonlinear Mobilities

Georg Heinze
TU Chemnitz

In this talk, the evolution of a system of two species with nonlinear mobility on a graph with nonlocal interactions is discussed. We provide a rigorous interpretation of the interaction system as a gradient flow in a Finslerian setting. This not only extends the recent results of Esposito et. al. (2021) to systems of interacting species, but also translates the theory of gradient flows with concave, nonlinear mobilities to this setting. Weakening the notion of Minkowski norm and nonlocal gradient, in the spirit of Agueh (2011), the geometric interpretations and the analysis are carried over to p-Wasserstein-like distances.

A Mean-Field Optimal Control Approach to the Training of NeurODEs

Hui Huang
University of Graz

In this talk, we consider a measure-theoretical formulation of the training of NeurODEs in the form of a mean-field optimal control with the L^2-regularization. We derive the first order optimality conditions for the NeurODEs training problem in the form of a mean-field maximum principle and show that it admits a unique control solution, which is Lipschitz continuous in time. Some instructive numerical experiments are also provided.

Surprising solutions to the ODE of constant mean curvature in homogeneous spaces

Philipp Käse
TU Darmstadt

In 1841 Delaunay described surfaces of constant mean curvature $H = 1$ in Euclidean space invariant under rotation: embedded unduloids and non-embedded nodoids. In 1989 Korevaar, Kusner, and Solomon showed that every embedded CMC cylinder (not assuming rotational invariance) must be a Delaunay unduloid. Therefore, there are no other embedded CMC cylinders. In the more general setting of homogeneous spaces this is surprisingly false: We can prove the existence of embedded CMC cylinders, which are not Delaunay unduloids.

Variational approaches to fluid-structure interaction

Malte Kampschulte
Charles University Prague

When considering problems in continuum mechanics, in particular those involving large deformations of a solid, one is generally faced with a highly non-linear, non-convex state-space of admissible solutions. As a result, many of the standard PDE-methods can only be applied after performing some problem-specific adaptations, if at all. In contrast to this, methods from the calculus of variations have no difficulties with those non-convexities, however they in turn cannot be easily applied to problems involving inertia, where the solutions do not have the structure of local minima. The aim of this contribution is to present a general approach to such kind of problems that relies on concepts from both sides and can be used to show existence of weak solutions for extremely general classes of complex physical problems. We illustrate this at an example from fluid-structure interaction. This is based on joint works with B. Benešová (Charles University), S. Schwarzacher (Uppsala).

Concentration-cancellation for the simplified Ericksen-Leslie model

Joshua Kortum
University of Würzburg

The Ericksen-Leslie system models the phase of nematic liquid crystals. It consists of a Navier-Stokes-like momentum equation and an extended harmonic map flow equation. Due to this coupling and the critical regime in two dimensions, the existence theory of solutions to such a system is non-trivial. We show that the set of approximative solutions to the Ericksen-Leslie system, in particular those arising from the Ginzburg-Landau approximation, is compact in the weak topology. The verification of the limit passage is executed by using concentration cancellation arguments introduced by DiPerna and Majda for the stationary Euler equations.

A viscous ergodic problem in noncompact setting: HJB equation and MFG

Hicham Kouhkouh
RWTH Aachen

We address the problem of existence of solutions (C, u) where C is a constant and u is a function, to ergodic Hamilton-Jacobi-Bellman equations in the whole space with unbounded and merely measurable ingredients. The method we use is different from classical approaches. It relies on duality theory and optimization in abstract Banach spaces together with maximal dissipativity of diffusion operators. The results also extend to ergodic mean-field games in the same setting.

A dynamic approach to heterogeneous elastic wires

Leonie Langer
Ulm University

We model heterogeneous elastic wires by closed planar curves with density. The associated elastic energy depends on a density-modulated stiffness. Working with an inclination angle function, the L^2-gradient flow corresponding to this energy is a nonlocal quasilinear coupled parabolic system of second order. We show local well-posedness and global existence of solutions and study properties of the flow. This talk is based on a joint work with Anna Dall'Acqua, Gaspard Jankowiak and Fabian Rupp.

Global solutions for fluid-elastic interaction with small data

Karoline Disser, Michelle Luckas
University of Kassel

In this talk, we consider a non-linear coupled system of the dynamical Lamé system and the Navier-Stokes equations modelling the dynamics of a linearly elastic body immersed in an incompressible viscous fluid without damping. We show the global existence of strong solutions for small data and also identify and discuss long-time asymptotic dynamics of the system. While the fluid velocity always approximates zero for large times, the long-time dynamics of the elastic part depend on the geometry of the considered domain.

Weak and stationary solutions to a cross-diffusion system with Cahn-Hilliard terms

Jean Cauvin-Vila[1], Virginie Ehrlacher[1], Greta Marino[2], Jan-Frederik Pietschmann[2]
INRIA Paris[1], Chemnitz University of Technology[2]

We study a Cahn-Hilliard model for a multicomponent mixture with cross-diffusion effects, degenerate mobility, and where only one of the species does separate from the others. We define a notion of weak solution adapted to possible degeneracies and our first result is (global in time) existence. In order to overcome the lack of a-priori estimates, our proof uses the formal gradient flow structure of the system and an extension of the boundedness by entropy method which involves a careful analysis of an auxiliary variational problem. This allows to obtain solutions to an approximate, time-discrete system. Letting the time step size go to zero, we recover the desired weak solution where, due to their low regularity, the Cahn-Hilliard terms require a special treatment. We then show existence of stationary solutions as critical points of the associated energy functional, and will provide some regularity properties.

This talk is based on joint works with Jean Cauvin-Vila and Virginie Ehrlacher (INRIA, Paris), and Jan-Frederik Pietschmann (TU Chemnitz).

Mean-field limit derivation of a gyrokinetic spray model

Matthieu Ménard
Université Grenoble Alpes

In this talk I will present a spray model describing an incompressible fluid and a density of solid particles interacting through a gyroscopic effect. I will focus on the mean-field limit derivation of the equations. Namely, I will consider a finite number of solid particles moving in a fluid and show that as the number of particle becomes large, the density of particles becomes very close to the solution of the spray model.

Elliptic equations involving surface measures

Marius Müller
University of Freiburg

In this talk we investigate the optimal regularity of Elliptic equations with a (Hausdorff-)measure-valued forcing term. I will try to convince you that Lipschitz regularity is best possible for these problems. Afterwards I will report under what (geometric) conditions on the forcing measure this Lipschitz regularity can be achieved.

Energy Minimizing Configurations for Highly Deformable Elastic Surfaces

Gokul Gopan Nair
Cornell University

We consider a class of nonlinearly elastic surfaces, modelled as immersions of a 2d domain in R^3 together with a vector field along the surface (referred to as the director). We require the director field to satisfy an orientation preserving condition, which is enforced by demanding that the energy density function (Lagrangian) at a point blows up as the director approaches the tangent plane at that point. By assuming coerciveness and physically reasonable convexity conditions for the energy density, we employ the direct method in the calculus of variations to establish the existence of global energy minimizers.

Kinetic maximal L^p-regularity

Lukas Niebel, Rico Zacher
Ulm University

In this talk, I want to give a short overview of the concept of kinetic maximal L^p-regularity and its applications. We are going to take a look at L^p-solutions to the Cauchy problem for the (fractional) Kolmogorov equation

$$\begin{cases} \partial_t u + v \cdot \nabla_x u + (-\Delta_v)^s u = f \\ u(0) = g \end{cases}$$

with $s \in (0, 2]$ and related linear equations. Then, I will briefly sketch how kinetic maximal L^p-regularity can be applied to study nonlinear kinetic equations. This is joint work with Prof. Dr. Rico Zacher, Ulm University.

Asymptotic behavior of minimizers for a singular perturbation of the one-dimensional Perona-Malik functional

Nicola Picenni
Scuola Normale Superiore

The topic of the talk is the study of minima and minimizers for an approximation and regularization of the Perona-Malik functional in dimension one, namely an integral functional whose Lagrangian is convex-concave with respect to the derivative, with a convexification that is identically zero. More precisely, the regularization is obtained by adding a term that depends on second order derivatives multiplied by a small coefficient, and we consider the limit when this parameter vanishes. In particular, it turns out that minimizers exhibit the so-called staircasing phenomenon, namely they develop a sort of microstructure that looks like a piecewise constant function at a suitable scale, in a rather strong sense. Moreover, the problem has a multi-scale nature, which is enlightened by some further results concerning the shape of minimizer at finer scales. This analysis relies on Gamma-convergence results for a rescaled functional, blow-up techniques, and a characterization of local minimizers for the limit problem. Finally, we discuss some possible extensions and some open questions that are related to this problem and to the generalization to higher space dimensions. This is a joint work with Massimo Gobbino.

The Willmore flow with Dirichlet boundary data

Manuel Schlierf
Ulm University

Evolution of immersed surfaces by the Willmore flow has been an active research topic for quite some while, sparked by early results of Kuwert-Schätzle. They proved global existence and convergence for spherical (especially: without boundary) immersions below a certain energy threshold. Later, Blatt constructed singular examples of spherical immersions arbitrarily close to the energy threshold whose Willmore flow does not converge. We in turn investigate the Willmore flow with prescribed Dirichlet boundary data for initial data which are surfaces of revolution and aim to find similar results.

Analysis of a soap film catenoid driven by an electrostatic force

Lina Sophie Schmitz
Leibniz University Hannover

A conductive soap film spanned between two parallel rings of equal radius takes the shape of a catenoid. Applying a voltage difference between this catenoid and an outer rigid cylinder barrel induces an electrostatic force deflecting the film. Its evolution can be modeled by a parabolic equation coupled with an elliptic equation for the electrostatic potential in the unknown domain between outer cylinder and soap film. For the rotational symmetric case, I will explain how this free boundary problem can be recast as a single parabolic equation with a non-local source term. I will also present results on existence and stability of stationary solutions.

Weakly Compressible Two-layer Shallow-Water Flows

Sarswati Shah, Gerardo Hernandez-Duenas
National University of Mexico

We present a weakly compressible approach to describe two-layer shallow water flows in channels with arbitrary cross sections [1, 2]. The standard approach for those flows results in a conditionally hyperbolic balance law with non-conservative products while the current model is unconditionally hyperbolic. A detailed description of the properties of the model is provided, including entropy inequalities and entropy stability. Furthermore, a high-resolution, non-oscillatory semi-discrete central-upwind scheme is presented. The scheme extends existing central-upwind semi-discrete numerical methods for hyperbolic balance laws. We also discuss the exact solution of the system and have shown the comparison with numerical results. Properties of the model such as positivity and well balance will be discussed.

References

[1] G. Hernandez-Duenas, J. Balbas, "A Central-Upwind Scheme for Two-layer Shallow-water Flowswith Friction and Entrainment along Channels", ESAIM: Mathematical Modelling and NumericalAnalysis, 55, pp. 2185–2210, 2021.

[2] A. Chiapolino, R. Saurel "Models and methods for two-layer shallow water flows", Journal of Computational Physics, 371, pp. 1043-1066, 2018.

Statistical solutions of the 2D incompressible Euler equations

Raphael Wagner
Ulm University

Statistical solutions generalize the concept of weak solutions of PDEs and are of particular interest when existence is known while uniqueness is unknown or even false. In the 1970s in the context of the 3D incompressible Navier-Stokes equations, important groundwork was laid by Foias and Prodi and by Vishik and Fursikov. In more recent years, both approaches were combined and certain methods of proof were generalized to make the theory more applicable for other equations. Within this framework, existence results of statistical solutions of the 2D Euler equations with vorticity in various Lebesgue spaces will be presented in the talk. Joint work with Emil Wiedemann.

Non-exchangeable interacting diffusions with singular kernels

Xianliang Zhao
University Bielefeld

We consider the mean-field limits of interacting diffusions without exchangeability, given rised by weighted interactions and non-i.i.d. initial values. The weights are of mixed signs and the interaction kernel, including the Biot-Savart law, enjoys L^p_q-type singularity. We prove general mean-field limits in the sense that any empirical measure process with uniform L^r-weights, $r > 1$, converges weakly to a coupled PDE. In particular, the empirical measure process is weighted in the same way as the interaction converges weakly to a nonlinear PDE.

Poster Session

- Johannes Bärlin
  Spectral stability of shock profiles in hyperbolically regularized systems of conservation laws
- Batuhan Bayır
  A Fourier Spectral Method for the Rosenau Equation
- Nicola De Nitti
  Liouville-type theorems for the 2D stationary MHD equations
- Francesco Esposito
  Symmetry results for singular problems
- Guy Foghem
  Stability of Nonlinear nonlocal problem driven by p-Lévy operators
- Ed Gallagher
  The infinity-elastica problem on a Riemannian manifold
- Georg Heinze
  Nonlocal cross-interaction systems on graphs
- Maren Hoberg
  Size-structured population models with spatial diffusion
- Malte Kampschulte
  Variational approaches to fluid-structure interaction
- Philipp Käse
  Surprising solutions to the ODE of constant mean curvature in homogeneous spaces
- Joshua Kortum
  Concentration cancellation for the simplified Ericksen-Leslie model
- Hicham Kouhkouh
  PDE and Control methods for global optimization and neural networks
- Marius Müller
  A biharmonic Bernoulli problem
- Gokul Gopan
  Elastic Membranes on Prescribed Surfaces

- Lukas Niebel
  Kinetic maximal L^p-regularity and applications to quasilinear equations
- T. Raja Sekhar
  Riemann problem and interaction of weak shocks in isentropic Cargo-LeRoux model of flux perturbation
- Manuel Schlierf
  The clamped Willmore flow for surfaces of revolution
- Kerstin Schmitz
  Convergence of a finite-volume scheme for a stochastic heat equation with nonlinear multiplicative noise
- Hyungeun Shin
  A posteriori error estimates of the Richards equation by the spatial and temporal flux reconstructions
- Sandeep Kumar Soni
  Analysis of Newtonian and non-Newtonian blood flow through multiple stenoses in a narrow artery
- Daniel Steenebrügge
  Making knots untangle themselves
- Raphael Wagner
  Statistical solutions of the incompressible Euler equations

List of Participants

Ayaz Ahmad (Patna)

Joannis Alexopoulos (Karlsruhe)

Wolfgang Arendt (Ulm)

Johannes Bärlin (Konstanz)

Batuhan Bayır (Istanbul)

Ivan Biocic (Zagreb)

Fabian Bleitner (Hamburg)

Patrick Brkic (Ulm)

David Buchberger (Augsburg)

José A. Carrillo (Oxford)

Anna Dall'Acqua (Ulm)

Francesco De Anna (Würzburg)

Nicola De Nitti (Erlangen-Nürnberg)

Mateusz Dembny (Warsaw)

Josef Demmel (Ulm)

Wolf-Patrick Düll (Stuttgart)

Ferdinand Eitler (Augsburg)

Hamid El bahja (Kapstadt)

Francesco Esposito (Arcavacata)

Alejandro Fernández-Jiménez (Oxford)

Julian Fischer (Klosterneuburg)

Guy Foghem (Dresden)

Mattia Freguglia (Pisa)

Mario Fuest (Leibniz)

Ed Gallagher (Bath)

Dennis Gallenmüller (Ulm)

Nibedita Ghosh (Kharagpur)

Florian Grün (Lausanne)

Jan Haskovec (Thuwal)

Georg Heinze (Chemnitz)

Maren Hoberg (Hannover)

Sarah Hofbauer (Stuttgart)

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Hui Huang (Graz)

Malte Kampschulte (Prague)

Philipp Käse (Damstadt)

Tobias Kielwein (Stuttgart)

Joshua Kortum (Würzburg)

Hicham Kouhkouh (Aachen)

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Christian Seis (Münster)

Sarswati Shah (Mexico City)

Hyungeun Shin (Prague)

Sandeep Kumar Soni (Zagreb)

Daniel Steenebrügge (Aachen)

Raphael Taraca (Stuttgart)

Parasuram Venkatesh (Mumbai)

Raphael Wagner (Ulm)

Emil Wiedemann (Ulm)

Pei Yee Woh (Hong Kong)

Christopher Wright (Oxford)

Rico Zacher (Ulm)

Xianliang Zhao (Bielefeld)

Map of the University