universität ulm

Elliptic and parabolic equations: interplay of asymptotics, regularity and geometry

Mini-Conference in honour of Tom ter Elst's 60th birthday

Organizers: Wolfgang Arendt, Manfred Sauter, Rico Zacher

Ulm, 19–21 June 2023

All talks take place at Ulm University, Helmholtzstr. 18, room 220.

(Last update: 2023-06-14)

1 Schedule

Monday, 19 June

8:45 Opening of conference

8:50–9:30 El Maati Ouhabaz
The harmonic lifting of elliptic operators with complex coefficients

9:30–10:10 Ralph Chill
Nonlinear nonlocal elliptic operators

Coffee break

10:40–11:20 Sylvie Monniaux
Keller-Segel Navier-Stokes system in non smooth domains

11:20–12:00 Gieri Simonett
On magnetoviscoelastic fluids in 3D

Lunch break

14:00–14:40 Hendrik Vogt
Non-autonomous semilinear parabolic equations and Schaefer's fixed point theorem

14:40–15:20 Jochen Glück
Maximal inequalities, kernel estimates, and regularity for C_0-semigroups

Coffee break

15:50–16:30 Malte Peter
Identification of microstructural information from macroscopic boundary measurements in steady-state linear elasticity

16:30–17:10 Clément Denis
An eigenvalue comparison theorem for the Stokes operator

18:30 Conference dinner at *Ristorante Italiano San Remo*

Tuesday, 20 June

9:10–9:50 Joachim Rehberg
The $\mathcal{H}^\infty$ angle for second order divergence operators on L^p

Coffee break

10:30–11:10 David Seifert
Energy decay in one-dimensional wave-heat systems

11:10–11:50 Robert Haller
The Kato square root property on locally uniformly domains

Lunch break

13:30–14:10 Dorin Bucur
Boundary behaviour of Robin problems in nonsmooth domains

14:10–14:50 Tim Binz
Well-Posedness of a Coupled Atmosphere-Sea Ice-Ocean Model

Taking public transport to Ulm City Centre

16:00–17:30 Guided tour at Ulm Minster
Roofs and Arches

Short stroll to Ulm City Library

18:00 Wolfgang Arendt and Manfred Sauter
Great walks and mathematics with Tom!

19:00 Reservation at Ulm's oldest restaurant *Krone Ulm*

Wednesday, 21 June

8:50–9:30 Charles Batty
Extensions of the Katznelson-Tzafriri theorem

9:30–10:10 Daniel Daners
The Logistic Equation on Rough Domains

Coffee break

10:40–11:20 Jussi Behrndt
The generalized Birman-Schwinger principle

11:20–12:00 James Kennedy
The spectrum of the Robin Laplacian with a complex parameter

Closing of conference

2 Abstracts

Extensions of the Katznelson-Tzafriri theorem

Charles Batty (University of Oxford)

In 1986, Katznelson and Tzafriri proved that, if T is a power-bounded operator on a Banach space X, and the spectrum of T meets the unit circle only at 1, then $\|T^n(I - T)\| \to 0$ as $n \to \infty$. Actually they went further and proved that $\|T^n f(T)\| \to 0$ if T and f satisfy certain conditions. Soon afterwards, analogous results were obtained for bounded C_0-semigroups $(T(t))_{t\geq 0}$. I will speak about those results, and in particular a recent result obtained by David Seifert and myself.

The generalized Birman–Schwinger principle

Jussi Behrndt (Technische Universität Graz)

In this lecture we discuss a generalized Birman–Schwinger principle in the non-selfadjoint context. In particular, we provide a detailed discussion of geometric and algebraic multiplicities of eigenvalues of the basic operator of interest (e.g., a Schrödinger operator) and the associated Birman–Schwinger operator, and additionally offer a careful study of the associated Jordan chains of generalized eigenvectors of both operators.

This talk is based on a joint paper with Fritz Gesztesy and Tom ter Elst.

Well-Posedness of a Coupled Atmosphere-Sea Ice-Ocean Model

Tim Binz (Technical University of Darmstadt)

This talk establishes local strong well-posedness and global strong well-posedness close to constant equilibria of a model coupling the primitive equations of the ocean and the atmosphere with a regularized version of Hibler's viscous-plastic sea ice model. Following the situation of the plane Couette flow, the ocean force on the sea ice is proportional to the shear rate and the velocity of the ocean and the ice coincide on their common interface. In addition, it is assumed that the atmosphere exerts a force on the sea ice via atmospheric winds.
To deal with the coupled system, we consider the stationary hydrostatic Stokes problem and study the associated Dirichlet and Dirichlet-to-Neumann operator. The latter operators are of independent interest.
Our strategy is to include the interface condition into the domain of the associated operator, which, however, then has non-diagonal domain.
A decoupling approach for this operator matrix using the above hydrostatic Dirichlet and Dirichlet-to-Neumann operator is presented.

Joint work with Felix Brandt and Matthias Hieber

T. Binz, F. Brandt, M. Hieber: *Coupling of the ocean and atmosphere dynamics with sea ice*. arXiv:2209.13150.

Boundary behaviour of Robin problems in nonsmooth domains

Dorin Bucur (Université Savoie Mont Blanc)

Let X be a Banach space. We consider the Laplace equation with Robin boundary conditions in a (nonsmooth) domain. Assume the right hand side is a bounded nonnegative function. Given a point on the boundary, the question is whether the solution has a strictly positive lower limit at this point. If the domain is smooth in a neighbourhood of the point (say of class C^2) the answer is positive, as a consequence of the Hopf maximum principle. If the domain is not smooth, the answer may be positive or negative, depending on the geometry of the domain around the point. This question was raised in a probabilistic context by Bass, Burdzy and Chen in 2008 where they give some answers covering, in particular, the case of Lipschitz sets and some cuspidal domains. Our motivation to analyze this problem comes from the regularity analysis of the free boundaries associated to shape optimization problems with Robin boundary conditions. In this talk, I will make the point on the question and I will present a new, variational, approach which allows us to deal with more general (nonlinear) elliptic operators and various geometric contexts, in particular with rectifiable boundaries of finite measure.

Joint work with A. Giacomini and M. Nahon.

Nonlinear nonlocal elliptic operators

Ralph Chill (TU Dresden)

We consider nonlocal elliptic operators on domains in $\mathbb{R}^N$, on Riemannian manifolds, on graphs or on random walk spaces with so-called Neumann, Robin or Dirichlet exterior conditions. The types of operators we consider are subgradients of convex energies, and in some special cases, they are "fractional powers" of Laplace or p-Laplace type operators. A main ingredient in the definition of these operators is an operator j, a restriction operator, which appeared first in several articles by Arendt and ter Elst. We show that the nonlocal operators considered here generate order preserving, L^∞-contractive semigroups, that is, that the associated energies are (nonlinear) Dirichlet forms.

Joint work with Mahamadi Warma (George Mason University).

The Logistic Equation on Rough Domains

Daniel Daners (University of Sydney, Australia)

We discuss the existence of steady state solutions of the abstract logistic equation

$$Au = \lambda u - m(x)g(u),$$

where $-A$ is the generator of a positive irreducible C_0-semigroup on $L^p(\Omega)$ with some smoothing properties. Here $\Omega \subseteq \mathbb{R}^N$ is a bounded domain, λ a real parameter, m a positive weight possibly vanishing on some subset of Ω and g a strictly increasing function with $g(0) = 0$. We prove the existence of a unique steady state solution for a suitable range of λ by means of the method of sub- and super-solutions. The key is the construction of a pair of ordered sub- and super-solutions that completely avoids the use of the Hopf boundary lemma that is often considered indispensible in the theory of semi-linear elliptic boundary value problems. As an alternative for such comparison theorems we use a cut-off weight m in conjunction with the Kato inequality that work in a abstract framework independent of any boundary conditions. The theory applies to the Dirichlet Laplacian on arbitrary bounded domains, the Robin Laplacian on almost arbitrary domains, and the Neumann Laplacian on Lipschitz domains.

Joint work with Wolfgang Arendt

References:

W. Arendt and D. Daners, *Semilinear eliptic equations on rough domains*, Journal of Differential Equations 346, 376–415, 2023.

An eigenvalue comparison theorem for the Stokes operator

Clément Denis (Université Aix-Marseille, France)

We prove an eigenvalue comparison theorem for the Stokes operator, inspired by Friedlander's theorem for the Laplacian. Using an abstract Dirichlet-to-Neumann graph, we prove the inequality $\lambda^N_{n+k} < \lambda^D_n$ where λ^D_j (respectively λ^N_j) is the j-th eigenvalues of the Dirichlet (respectively, Neumann) Stokes operator, and k is the number of negative eigenvalues for the associated Dirichlet-to-Neumann graph.

This is joint work with Tom ter Elst.

Maximal inequalities, kernel estimates, and regularity for C_0-semigroups

Jochen Glück (Bergische Universität Wuppertal)

Consider a C_0-semigroup $T = (T(t))_{t\in[0,\infty)}$ on an L^p-space which leaves the positive cone in L^p invariant. We present the following recent result and discuss its context: Assume that T satisfies a maximal inequality for small times, meaning that for every $f \in L^p$ there exists $h \in L^p$ such that $|T(t)f| \leq h$ for all $t \in [0,1]$. Then the semigroup T is analytic.

As a consequence one obtains that a positive C_0-semigroup on L^p is automatically analytic if it satisfies Gaussian bounds or similar kernel estimates.

The Kato square root property on locally uniformly domains

Robert Haller (Technical University Darmstadt)

The famous Kato square root property states that for second order elliptic operators in divergence form, defined on L^2 via the corresponding sesquilinear form, the domain of its square root coincides with the form domain. Since the breakthrough by Auscher, Hofmann, Lacey, M$^{\text{c}}$Intosh, and Tchamitchian in 2001 it was established in more and more general situations.

This talk will focus on the case of spatial domains with non-smooth boundary and mixed boundary conditions. I want to describe how during the last decade the conditions needed to prove the Kato property in this setting could be reduced to just two: The Dirichlet part has to be $(n-1)$-dimensional in an adequate sense and near the complementary boundary the domain has to fulfill some quantitative connectivity condition in the spirit of locally uniform domains.

Joint work with Sebastian Bechtel, Moritz Egert and Patrick Tolksdorf

The spectrum of the Robin Laplacian with a complex parameter

James Kennedy (University of Lisbon)

We discuss properties of the Robin Laplacian on a bounded Lipschitz domain Ω, where the parameter α appearing in the boundary condition

$$\frac{\partial u}{\partial \nu} + \alpha u = 0$$

on $\partial\Omega$ (ν being the outward-pointing unit normal to Ω) is complex and hence the operator is no longer self-adjoint.

Besides giving an overview of basic well-posedness and generation properties of the operator, we will be interested in the behaviour of the spectrum and the numerical range as functions of α, in particular as $\alpha \to \infty$ in $\mathbb{C}$. The spectrum consists of complex eigenvalue curves which are (possibly multivalued) meromorphic functions of α, and which also enjoy a duality with the (complex) Dirichlet-to-Neumann operator (for the real case see [1,2]; the complex operator is treated in [3]).

We will give sharp trace-type inequalities which show that the numerical range must lie in a parabolic region in $\mathbb{C}$; this, in particular, leads to information about the potential rate of divergence of the eigenvalues as $\alpha \to \infty$, including eigenvalue bounds which are new even in the case of real α. We will also discuss the nature of convergence and/or divergence of the eigenvalues in dependence on the ratio between $\operatorname{Re}\alpha$ and $\operatorname{Im}\alpha$ as $\alpha \to \infty$.

This talk is based primarily on joint work [4] with Sabine Bögli (Durham) and Robin Lang (Stuttgart).

References:

1. W. Arendt and A. F. M. ter Elst, *The Dirichlet-to-Neumann operator on rough domains*, J. Differential Equ. 251 (2011), 2100–2124.

2. J. Behrndt and A. F. M. ter Elst, *Dirichlet-to-Neumann maps on bounded Lipschitz domains*, J. Differential Equ. 259 (2015), 5903–5926.

3. J. Behrndt and A. F. M. ter Elst, *Jordan chains of elliptic partial differential operators and Dirichlet-to-Neumann maps*, J. Spectr. Theory 11 (2021), 1081–1105.

4. S. Bögli, J. B. Kennedy and R. Lang, *On the eigenvalues of the Robin Laplacian with a complex parameter*, Anal. Math. Phys. 12 (2022), 39.

Keller-Segel Navier-Stokes system in non smooth domains

Sylvie Monniaux (Université Aix-Marseille, France)

We are interested in the Keller-Segel Navier-Stokes system which describes a cell population with a chemical attractant in a fluid in dimension 3, in a Lipschitz domain. The unknowns of the system are the density of the cell population, the concentration of the chemical attractant and the velocity (and the pressure) of the fluid. One can determine critical spaces for each of these unknowns and prove existence of solutions in these critical spaces. The tools are weighted maximal regularity and recent results on the Dirichlet-Stokes system in Lipschitz domains.

This is a joint work with Matthias Hieber (TU Darmstadt), Hideo Kozono (Waseda Univ. Tokyo) and Patrick Tolksdorf (KIT, Karlsruhe).

The harmonic lifting of elliptic operators with complex coefficients

El Maati Ouhabaz (University of Bordeaux)

We consider a divergence form elliptic operator of second order L on a bounded domain Ω. It is a standard fact that if the boundary Γ is Lipschitz, then one can solve

$$Lu = 0 \text{ in } \Omega, \quad \mathrm{Tr}(u) = \phi \text{ on } \Gamma$$

for $\phi \in H^{1/2}(\Gamma)$. It is of interest to decide whether the harmonic lifting $\phi \mapsto u$ extends to a bounded operator from $L^p(\Gamma)$ into $L^p(\Omega)$. We discuss new results in the case where the coefficient of L are complex and satisfy a smoothness assumption.

This talk is based on a joint work (in progress) with Tom ter Elst.

Identification of microstructural information from macroscopic boundary measurements in steady-state linear elasticity

Malte A. Peter (Universität Augsburg)

We consider the upscaled linear elasticity problem in the context of periodic homogenization. Based on measurements of the deformation of the (macroscopic) boundary of a body for a given forcing, it is the aim to deduce information on the geometry of the microstructure. For a parametrized microstructure, we are able to prove that there exists at least one solution of the associated minimization problem based on the L^2-difference of the measured deformation and the resulting deformation for a given parameter. To facilitate the use of gradient-based algorithms, we derive the Gâteaux derivatives using the Lagrangian method of Céa, and we present numerical experiments showcasing the functioning of the method.

This is joint work with T. Lochner (University of Augsburg).

The $\mathcal{H}^\infty$ angle for second order divergence operators on L^p

Joachim Rehberg (Weierstrass Institute, Berlin)

For decades people believed that $\arctan \frac{M}{m}$ is the adequate estimate for the sector (half)angle α of the numerical range of a Dirichlet form – M being the bound for the coefficient function and m its ellipticity constant. We show that α can be improved to $\arctan \sqrt{\frac{M^2}{m^2} - 1}$ – which turns out to be optimal then in general.
Numerous enjoyable consequences for the elliptic and parabolic theory of the induced operators on L^2 and even in the whole L^p scale are presented. In particular, this affects the $\mathcal{H}^\infty$ angle for the corresponding operators.

Joint work with Tom terElst and Alexander Linke

A. F. M. ter Elst, A. Linke, J. Rehberg: *On the numerical range of sectorial forms*. Pure Appl. Funct. Anal. 7, No. 5, 1929-1938 (2022).

Energy decay in one-dimensional wave-heat systems

David Seifert (Newcastle University)

We begin by considering the coupled wave-heat system

$$\begin{cases} u_{tt}(x,t) = u_{xx}(x,t), & x \in (-1,0),\ t > 0, \\ w_t(x,t) = w_{xx}(x,t), & x \in (0,1),\ t > 0, \end{cases}$$

subject to suitable initial conditions, mixed boundary conditions $u_x(-1,t) = w(1,t) = 0$ for all $t > 0$ and appropriate coupling conditions at the interface $x = 0$. This may be viewed as a simple linear model of fluid-structure interaction in one spatial dimension. By reformulating the system as an abstract Cauchy problem on a suitable Hilbert space, I will explain why it is well posed and outline a proof using the Borichev–Tomilov theorem that the energy of any classical solution must decay at the rate t^{-4} as $t \to \infty$. Moreover, this decay rate is sharp.

In the second part of the talk I will turn to a closely related system in which the heat equation is modified according to the Coleman–Gurtin thermal law. By means of a more involved approach, using Dafermos' history space framework and ideas from systems theory, it can be shown that the energy decay rate remains the same, and again is optimal. I will comment on some of the key aspects here.

The first part of the talk is based on joint work with Charles Batty and Lassi Paunonen, the second part on more recent work with Filippo Dell'Oro and Lassi Paunonen.

On magnetoviscoelastic fluids in 3D

Gieri Simonett (Vanderbilt University)

In this talk, we consider a system of equations that model a non-isothermal magnetoviscoelastic fluid which is thermodynamically consistent. The system is investigated in the framework of quasilinear parabolic systems. First, we will discuss local existence and uniqueness of strong solutions. Then it will be shown that solutions that are initially close to a constant equilibrium exist globally and converge to a (possibly different) constant equilibrium. Finally, we show that that every solution that is eventually bounded in the topology of the state manifold exists globally and converges to the set of equilibria.

Joint work with Hengrong Du and Yuanzhen Shao

H. Du, Y. Shao, G. Simonett *On a thermodynamically consistent model for magnetoviscoelastic fluids in 3D*. ArXiv:2305.13432

Non-autonomous semilinear parabolic equations and Schaefer's fixed point theorem

Hendrik Vogt (Universität Bremen)

We show the existence of solutions of nonlinear non-autonomous Cauchy problems

$$\partial_t u(t,x) - \nabla_x \cdot (a(t,x)\nabla_x u(t,x)) = f(t,x,u(t,x),\nabla u(t,x)), \qquad u(0,\cdot) = u_0$$

for a bounded open set $\Omega \subseteq \mathbb{R}^n$. The coefficient matrix a is supposed to be symmetric, uniformly elliptic, Lipschitz continuous w.r.t. $t \in (0,\tau)$ and measurable w.r.t. $x \in \Omega$; the nonlinearity f is required to be continuous and to satisfy a suitable growth condition. We show that, given $u_0 \in H_0^1(\Omega)$, there exists $u \in L_2(0,\tau;H_0^1(\Omega)) \cap H^1(0,\tau;L_2(\Omega))$ solving the problem mentioned above.

The proof relies on Schaefer's fixed point theorem. In the course of the proof one uses maximal regularity properties of solutions of inhomogeneous linear problems and compact embeddings of vector-valued Sobolev spaces.

The result is inspired by [ArCh10].

The talk is based on joint work with Wolfgang Arendt and Jürgen Voigt.

[ArCh10] W. Arendt, R. Chill: *Global existence for quasilinear diffusion equations in isotropic nondivergence form*. Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) **IX**, 523–539 (2010).