Evolutionsgleichungen in Ulm 2018

Workshop zur Seniorprofessur von Wolfgang Arendt

# Liste der Titel und Abstracts

## **Ralph Chill** (Technische Universität Dresden)

### *Eine positive Welt*

(Hauptvortrag, ohne Abstract)

## **Markus Haase** (Christian-Albrechts-Universität zu Kiel)

### *Einige Bemerkungen zum Satz von Gelfand*

Der Satz von Gelfand (auch “kommutativer Satz v. Gelfand–Naimark” genannt) ist eines der zentralen Resultate der Funktionalanalysis. Wir beleuchten einige vielleicht weniger bekannte Aspekte des Satzes und erörtern die Frage, ob und wozu man ihn überhaupt benötigt.

## **Daniel Hauer** (University of Sydney)

### *Non-concavity of the Robin ground state*

On a convex bounded Euclidean domain, the ground state for the Laplacian with Neumann boundary conditions is a constant, while the Dirichlet ground state is log-concave. The Robin eigenvalue problem can be considered as interpolating between the Dirichlet and Neumann cases, so it seems natural that the Robin ground state should have similar concavity properties. In this talk, I show that this is false, by analysing the perturbation problem from the Neumann case. In particular, I prove that on polyhedral convex domains, except in very special cases (which we completely classify) the variation of the ground state with respect to the Robin parameter is not a concave function. One can conclude from this that the Robin ground state is not log-concave (and indeed even has some superlevel sets which are non-convex) for small Robin parameter on polyhedral convex domains outside a special class, and hence also on arbitrary convex domains which approximate these in Hausdorff distance.

This is a joint work with Ben Andrews (ANU, Canberra) and Julie Clutterbuck (Monash University).

**Matthias Hieber** (Technische Universität Darmstadt)

*From the Arendt–Bu Theorem to the Bidomain Equations and Backwards*

The Arendt–Bu Theorem characterizes the existence of periodic solutions to linear evolution equations in terms of the $R$-boundedness of the resolvent of the underlying generator. In this talk, we develop semi-and quasilinear versions of this theorem and apply them to the bidomain equations. We show that the associated bidomain operator generates an analytic semigroup within the $L^p$-setting having many interesting properties. They allow, combined with the Arendt–Bu Theorem, to prove the existence of periodic solutions to the nonlinear bidomain equations.

**James Kennedy** (Universidade de Lisboa)

*(Nicht nur) Diffusion bestimmt die Form des Gebiets*

Vor gut 50 Jahren formulierte Mark Kac seine berühmt-berüchtigte Frage, ob man die Form einer Trommel hören könne, das heißt, ob die Existenz eines unitären Operators, der zwei Dirichlet-Laplaceoperatoren verflicht, impliziert, dass die zugehörigen Gebiete isometrisch sind. Trotz einer negativen Antwort in Form Anfang der 1990er Jahre erschienener Gegenbeispiele gelang es Wolfgang Arendt wenige Jahre später in einer wegweisenden Arbeit, zu zeigen, dass die Antwort positiv wird, wenn man den "unitären Operator" durch einen "Ordnungsisomorphismus" ersetzt. Daraus kann man herleiten, dass Diffusion das Gebiet bestimmt: keine zwei Gebiete besitzen die "gleiche" Wärmeleitungshalbgruppe.

Wir schildern den Kern seines Arguments und zeigen, dass dieses unter deutlich schwächeren Voraussetzungen seine Gültigkeit behält: im Wesentlichen reicht es, wenn der verflechtende Operator *disjunktheitserhaltend* ist. Diese Eigenschaft scheint besonders natürlich im Angesicht der bekannten Gegenbeispiele zu Kac' ursprünglicher Frage, denn jedes besteht aus einem Paar Gebiete, die genau durch eine (endliche) Überlagerung lokaler Isometrien aufeinander abgebildet werden. Unsere Ergebnisse beruhen auf einer laufenden Zusammenarbeit mit Wolfgang Arendt.

**Markus Kunze** (Universität Konstanz)

*Convergence of sesquilinear forms and some applications to mathematical biology*

We present a convergence theorem for not necessarily densely defined sesquilinear forms due to Simon and Ouhabaz. We apply this result to establish an averaging principle for fast diffusions in domains separated by semi-permeable membranes. We also discuss some applications to mathematical biology. This is based on joined work with Adam Bobrowski and Bogdan Kazmierczak.

## **Sylvie Monniaux** (Université Aix-Marseille)

### *Compactness of the trace operator*

If the bounded domain $\Omega \subset \mathbb{R}^d$ has a Lipschitz boundary, it is known that the space

$$X_T := \left\{u \in L^2(\Omega,\mathbb{R}^d) : \operatorname{div} u \in L^2(\Omega),\ \operatorname{curl} u \in L^2(\Omega,\mathbb{R}^{d\times d}) \text{ and } \nu \cdot u = 0 \text{ on } \partial\Omega\right\}$$

where $\nu$ denotes the exterior unit normal vector at $\partial\Omega$ (defined almost everywhere) is not contained in $H^1(\Omega,\mathbb{R}^d)$. The optimal embedding, proved recently by M. Costabel, is that $X_T \subset H^{1/2}(\Omega,\mathbb{R}^d)$. It is however also known that the trace operator (on $\partial\Omega$) from $X_T$ to $L^2(\partial\Omega,\mathbb{R}^d)$ is bounded. An open question, to which I propose to give a positive answer, is whether this operator from $X_T$ to $L^2(\partial\Omega,\mathbb{R}^d)$ is compact.

## **Rainer Nagel** (Universität Tübingen)

### *Von Arendt–Greiner-Gruppen zu gewichteten Koopman-Halbgruppen*

In den 1980er Jahren haben W. Arendt und G. Greiner $C_0$ Gruppen positiver Operatoren auf Banachverbänden untersucht und dafür spektrale Abbildungssätze und Zerlegungssätze bewiesen. Ich werde zeigen, wie eine passende Koopmanlinearisierung differenzierbarer Flüsse auf Mannigfaltigkeiten zu analogen Resultaten führt.

## **Abdelaziz Rhandi** (Università degli Studi di Salerno)

### *$L^p$-theory for Schrödinger systems*

In this talk we study for $p \in (1,\infty)$ the $L^p$-realization of the vector-valued Schrödinger operator $\mathscr{L}u := \operatorname{div}(Q\nabla u) + Vu$. Using a noncommutative version of the Dore–Venni theorem due to Monniaux and Prüss, we prove that $L_p$, the $L^p$-realization of $\mathscr{L}$, defined on the intersection of the natural domains of the differential and multiplication operators which form $\mathscr{L}$, generates a strongly continuous contraction semigroup on $L^p(\mathbb{R}^d;\mathbb{C}^m)$. We also study additional properties of the semigroup such as positivity, ultracontractivity, Gaussian estimates and compactness of the resolvent. We end the talk by giving several examples and counterexamples.

This is a joint work with Markus Kunze, Luca Lorenzi and Abdallah Maichine.

## **Tom ter Elst** (University of Auckland)

### *The Dirichlet problem without the maximum principle*

Consider the Dirichlet problem with respect to an elliptic operator

$$A = -\sum_{k,l=1}^{d} \partial_k \, a_{kl} \, \partial_l - \sum_{k=1}^{d} \partial_k \, b_k + \sum_{k=1}^{d} c_k \, \partial_k + c_0$$

on a bounded Wiener regular open set $\Omega \subset \mathbb{R}^d$, where $a_{kl}, c_k \in L_\infty(\Omega, \mathbb{R})$ and $b_k, c_0 \in L_\infty(\Omega, \mathbb{C})$. Suppose that the associated operator on $L_2(\Omega)$ with Dirichlet boundary conditions is invertible. Then we show that for all $\varphi \in C(\partial\Omega)$ there exists a unique $u \in C(\overline{\Omega}) \cap H^1_{\text{loc}}(\Omega)$ such that $u|_{\partial\Omega} = \varphi$ and $Au = 0$.

In the case when $\Omega$ has a Lipschitz boundary and $\varphi \in C(\overline{\Omega}) \cap H^{1/2}(\overline{\Omega})$, then we show that $u$ coincides with the variational solution in $H^1(\Omega)$.

This is joint work with Wolfgang Arendt.

## **Mahamadi Warma** (University of Puerto Rico)

### *Analysis of the null Controllability of space-time fractional diffusion equations*

We consider the following class of fractional partial differential equations of evolution in which two parameters are used to sharpen the models.

$$\begin{cases} \mathbb{D}_t^\alpha u(t,x) + (-\Delta)^s u(t,x) = f(t,u) \quad \text{on } \Omega \times (0,T), \\ + \text{Intial conditions,} \\ + \text{Boundary conditions.} \end{cases}$$

Here $T > 0$ is a fixed time, $0 < \alpha \leq 2$, $0 < s \leq 1$, $\Omega \subset \mathbb{R}^N$ is an open set with boundary $\partial\Omega$, $(-\Delta)^s$ is the fractional Laplace operator and $\mathbb{D}_t^\alpha$ denotes a time fractional derivative. After clarifying which initial and boundary conditions make the system well posed, we show what is so far known about the null controllability or/and the approximate controllability of the above system. We conclude by given several open problems. The talk will be delivered for a wide audience avoiding unnecessary technicalities.