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Financial Mathematics II

Exercise Sheet 9

Discussion: Thursday 02/07/2015, 16:00-17:30, He18, E60,
and Friday 03/07/2015, 08:15-09:45, He18, 120.
Handing in: Thursday 25/06/2015, beginning of the lecture.

Exercise 9.1

Let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}, P)$ be a stochastic basis satisfying Convention 5.9 and $X = (X_t)_{t\in[0,T]}$ be a semimartingale. For $\varphi \in \mathbb{L}$ define

$$Y_t = \int_0^t \varphi_s \,\mathrm{d}X_s\,, \quad t \in [0,T]\,.$$

Show that then $Y = (Y_t)_{t\in[0,T]}$ is a semimartingale and it holds for $\psi \in \mathbb{L}$

$$\int_0^t \psi_s \,\mathrm{d}Y_s = \int_0^t \psi_s \,\mathrm{d}\left(\int_0^s \psi_u \,\mathrm{d}X_u\right) = \int_0^t \psi_s\varphi_s \,\mathrm{d}X_s\,.$$

Exercise 9.2

Let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}, P)$ be a stochastic basis satisfying Convention 5.9. Let $((X,Y)_t)_{t\in[0,T]}$ be a bivariate standard Brownian motion. Let $0 < \alpha < 1$ and define

$$B_t = \alpha X_t + \sqrt{1-\alpha^2}Y_t\,, \quad t \in [0,T]\,.$$

Then $(B_t)_{t\in[0,T]}$ is a univariate standard Brownian motion. Calculate $[B,B]$, $[X,Y]$, $[X,B]$, and $[Y,B]$.

Exercise 9.3

Let $(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F}_t)_{t\in[0,T]}, P)$ be a stochastic basis satisfying Convention 5.9, X, Y be semimartingales, and $H \in \mathbb{D}_{[0,T]}$. Let $\pi_n = \{0 = \tau_{1,n} \le \tau_{2,n} \le \cdots \le \tau_{m(n),n} = T\}$, $n \in \mathbb{N}$, be a sequence of stopping times with

$$\lim_{n\to\infty} \sup_{i=1,\dots,m(n)-1} |\tau_{i+1,n} - \tau_{i,n}| = 0 \quad \text{a.s.}$$

Show that then

$$\sum_{i=1}^{m(n)-1} H_{\tau_{i,n}}(X_{\tau_{i+1,n}\wedge t} - X_{\tau_{i,n}\wedge t})(Y_{\tau_{i+1,n}\wedge t} - Y_{\tau_{i,n}\wedge t}) \xrightarrow{\text{ucp}} \int_0^t H_{s-}\,\mathrm{d}[X,Y]_s\,, \quad \forall\, t \in [0,T]\,.$$