Computational Finance - Excercise Sheet 2

Exercise 1 Apply Itô's Formula to show that

$$Y_t = e^{\lambda W_t - \frac{1}{2}\lambda^2 t}$$

solves $dY_t = \lambda Y_t dW_t$.
Hint: Set $X_t := W_t$ and introduce an appropriate function $f(x,t)$.

Exercise 2 Implement Algorithm III. 2.1.2 for

(i) a European call with $S_0 = 5, K = 7, r = 0.06, \sigma = 0.3, M = 50, T = 1$.

(ii) a European digital option given by the payoff function $g(S) = 1_{S>K}$ with same variables as in (i).

What convergence rates do you observe with respect to the number of time steps M? (Choose $M = 2^k, k = 4, \ldots, 9$ and compare the solutions of the binomial model with the one deduced by the Black-Scholes formula

$$\begin{aligned} C_t^{\text{Call}} &= S_t \Phi(d_1(t)) - K e^{-r(T-t)} \Phi(d_2(t)) \\ d_1(t) &= \frac{\ln(\frac{S_t}{K}) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \\ d_2(t) &= d_1(t) - \sigma\sqrt{T-t} \end{aligned}$$

and $C_t^{\text{dig}} = e^{-rt}\Phi(d_2(t))$.)

Exercise 3 Adapt Algorithm III. 2.1.2 for American options.