Computational Finance - Excercise Sheet 3

Exercise 1 Apply Itô's Formula to show that the option price value follows the SDE stated in the lecture.

Exercise 2 Apply the Feynman-Kac Theorem on the Black-Scholes model to derive the Black-Scholes equation. Use that the price under risk-neutral valuation follows the SDE

$$dS_t = rS_t dt + \sigma S_t dW_t,$$

for $r > 0$ being the riskfree interest rate. Further use that the value of the option in t is given by $e^{-r(T-t)}\mathbb{E}(V(T, S(T))|S_t = s)$.

Exercise 3 The option price in the Black-Scholes market model depends on the parameters S_0, K, T, r and σ. Of these, σ is not observable at the market. One can use the observed option price V_{market} to infer the so-called implied volatility by solving $V(\sigma) = V_{\text{market}}$ for σ. We can observe that σ is not constant, as assumed in the Black-Scholes model. Therefore, one looks at the implied volatility surface, i.e. the implied volatility as a function of strike K and maturity T, $\sigma = \sigma(K, T)$.

Suppose that the Google Inc. stocks are traded at 526.88$. The European call options on Google are priced as in the table below. The 1-year US Treasury Bills rate is $r = 0.11\%$.

		Maturity	T		
Strike K	0.00555	0.0889	0.17222	0.3388	0.75
450	78.8	78.2	80.4	82.9	89.7
480	48.8	49.8	53.3	57.7	66.4
500	28.8	32.7	37.4	43.0	53.1
510	20.3	24.8	30.4	36.6	47.2
520	12.2	18.1	24.3	30.8	41.5
530	6.5	12.7	19	25.5	36.6
550	1.3	5.5	10.9	17	27.3
570	0.35	1.95	5.8	10.8	20.2
600	0.25	0.45	2	5.1	21.4

Use the data to construct an empirical volatility surface for Google Call options by computing the implied volatility at the different data points.

Hint: Use the MATLAB function `fzero` on the function $f(\sigma) = V(\sigma) - V_{\text{market}}$ to compute the implied volatility σ. For $V(\sigma)$ use the Black-Scholes formula stated on Exercise Sheet 2.