Exercise 1 Implement a function that computes the Euler-Maruyama approximation of a process
dX_t = a(t, X_t)dt + b(t, X_t)dW_t
for a given Wiener process path W_t, a maturity T, an initial value X_0 and functions a(t, x) and b(t, x) (you can pass functions using a function handle). Use this function to compute for the GBM

dX_t = 2X_t dt + X_t dW_t

- the absolute error at the endpoint T,
- the error of the entire process path.

Plot the error of different discretisation levels. Which convergence rate do you expect? What do you observe?

Exercise 2

- Use the Euler Maruyama scheme to compute the option price of the European Put with a Monte-Carlo simulation (S_0 = 20, K = 25, r = 0.02, \sigma = 0.4, T = 1.5).
- Adapt your program to price a European Up& Out Put (payoff function given by \( V_T = (K - S_T)^+ 1_{S_t < H t \in [0, T]} \)). Compute the fair price of such an option with parameters as in (i) and \( H = 23 \).

Which convergence rates do you observe w.r.t. the time discretisation?