

Computational Finance - Exercise Sheet 6

Exercise 1 Implement a function that computes the Euler-Maruyama approximation of a process

$$dX_t = a(t, X_t)dt + b(t, X_t)dW_t$$

for a given Wiener process path W_t , a maturity T , an initial value X_0 and functions $a(t, x)$ and $b(t, x)$ (you can pass functions using a function handle). Use this function to compute for the GBM

$$dX_t = 2X_tdt + X_tdW_t$$

- the absolute error at the endpoint T ,
- the error of the entire process path.

Plot the error of different discretisation levels. Which convergence rate do you expect? What do you observe?

Exercise 2

- Use the Euler Maruyama scheme to compute the option price of the European Put with a Monte-Carlo simulation ($S_0 = 20, K = 25, r = 0.02, \sigma = 0.4, T = 1.5$).
- Adapt your program to price a European Up& Out Put (payoff function given by $V_T = (K - S_T)^+ 1_{S_t < H \forall t \in [0, T]}$). Compute the fair price of such an option with parameters as in (i) and $H = 23$.

Which convergence rates do you observe w.r.t. the time discretisation?