

Computational Finance - Exercise Sheet 10

Exercise 1 The function $f(x) = |x|$, $f : (-1, 1) \rightarrow \mathbb{R}$ is not differentiable in the classical sense. Show that it is a function in $H^1(-1, 1)$, i.e. derive its weak derivative.

Exercise 2 Recall Definition IV 2.2.9. Derive the stated weak formulation starting at $(f, \varphi)_0 = (-\Delta u, \varphi)_0 = \dots$ for $\varphi \in C_0^\infty(\Omega)$.

Exercise 3 Given the pseudo code:

Require: solution guess u_0 , $tol = 10^{-6}$

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1:  $err = 1000, k = 0, u = u_0$ 
2: while  $err > tol$  do
3:   for  $i = 1 : n$  do
4:     for  $j = 1 : n$  do
5:        $u(i, j) = ((u(i-1, j) + u(i+1, j) + u(i, j-1) + u(i, j+1)) - h^2 * f(i, j))/4$ 
6:     end for
7:   end for
8:    $err = \max_{i,j} |u(i, j) - u_0|$ 
9:    $u_0 = u$ 
10:   $k = k + 1$ 
11: end while
12: return  $u$ 
    
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- (i) What kind of problem is to be solved? Write down the equation that is solved.
- (ii) Comment the algorithm, i.e. explain for every line what the program is doing. How does the solution is approached?
- (iii) What information have to be added to actually run that code? Where has this information to be added in the code? (State the line before and after.)
- (iv) Assume you implemented the algorithm. When you look at the figure, what happens? What should be added to the code for cases like that?

