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Institute of Mathematical Finance

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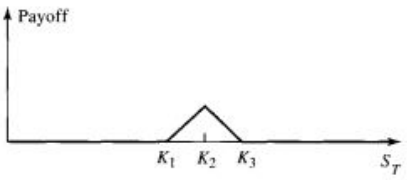
Financial Mathematics -DAV Supplement

Exercise Sheet

Exercise 1

Let us consider the following payoff diagram:



Find the formal representation of the payoff (using calls/puts). Here $K_2 = \frac{1}{2}(K_1 + K_3)$

Exercise 2

Let us consider the forward contract on buying one unit of the stock S with expiry date $T = 180$ days (1 year $\widehat{=}$360 days) and with the delivery price $K = 28$. Assume that until the expiry date no dividend is paid on this stock. Moreover, let $S(0) = 25$ and assume that the continuous rate of a bank account is equal to $r = 10\%$ per year.

a) Show that there exist an arbitrage opportunity on this market.

b) Find an arbitrage strategy.

c) For which K is there no possibility to achieve a riskless profit?

Exercise 3

Let us consider a long position in a forward contract on buying one unit of the stock with expiry date $T = 90$ days (1 year $\widehat{=}$360 days). Assume that until the expiry date no dividend is paid on this stock. Assume that the current value of the stock is equal to $S(0) = 40$ Euro. Assume that the continuous rate of a bank account is equal to $r = 5\%$ per year.

a) Find a T- Forward-price at time $t = 0$. What is the value of the future at $t = 0$?

b) Assume that we have this forward contract with a delivery price $K = 43$ EUR. How much should we (long position) pay for this contract?

Exercise 4

A company entered into a forward contract 3 months ago in order to buy a stock S. The remaining term of this forward contract is 100 days and the delivery price of the contract is $50,25$ EUR. For some reasons the company does not need this stock any more and enters into a new forward contract in order to sell this stock in 100 days. At this moment the value of the stock is $S(0) = 45$ EUR and the riskless continuous interest rate $r = 4.75\%$ per year (1 year $\widehat{=}$360 days).

a) Calculate the T-Forward price (delivery price) of the new contract.

b) Calculate the overall position at maturity date of both contacts.

c) What is the value of this overall position today?

Exercise 5

Assume that a company wants to hedge its portfolio at time $t = 1$ by selling a future contract. The portfolio consists of 100 units of a stock S. Today's price of the stock is $S(0) = 100$ EUR. The variance of the stock price at $t = 1$ equals 100. At any time the money can be put on or withdrawn from a bank account with the riskless (continuous) interest rate $r = 4\%$. All future contracts considered below have maturity 1 and consist of one unit of the suitable underlying. All Marking-to-Market effects are not taken into consideration.

a) Assume first that the stock S is also the underlying of the future contract. Calculate the T-Future price at time $t = 0$.

b) Assume that on the market there exist only futures on the underlying $\tilde{S}$. The current value of the underlying $\tilde{S}$ is equal to 200 EUR, the variance at time $t = 1$ equals 400 (i.e. $\mathrm{Var}(\tilde{S}(1)) = 400$) and $\mathrm{Cov}(S(1), \tilde{S}(1)) = 180$. Give the number of Futures in the minimal variance hedge.

Exercise 6

Assume that a company wants to hedge its portfolio at time $t = 1$ by selling a future contract. The portfolio consists of 100 units of a stock S. The current price of the stock is $S(0) = 150$ EUR. The variance of the stock price at $t = 1$ equals 60. On the market there exist only futures on the underlying $\tilde{S}$. The current value of the underlying $\tilde{S}$ is equal to 230 EUR, the variance at time $t = 1$ equals 500 and $\mathrm{Cov}(S(1), \tilde{S}(1)) = 220$. At any time the money can be put on or withdraw from a bank account with the riskless interest rate $r = 4\%$. All future contracts considered below have maturity 1 and consist of one unit of the suitable underlying. All Marking-to-Market effects are not taken into consideration.

a) How many futures should one sell in the minimal variance hedging?

b) How many futures should one sell in the minimal variance hedging in case that maturity $T = 4$?

Exercise 7

Let consider a (spot-starting) receiver-swap with the fixed interest rate $K = 5\%$, the face value $N = 10$ Mio. , maturity 3 years and annual payment. The continuous Spot rate curve today at time $t = 0$ is given by

T	1	2	3	4	5
R(0,T)	4,62%	4,65%	4,70%	4,75%	4,79%

a) Calculate the value of swaps and the "faire" swap rate $S_{0,3}(0)$.

b) Calculate the value of 3x2-swaps (i.e. the forward-swaps with $T_0 = 3, T_1 = 4$ and $T_2 = 5$) with $K = 5\%$, $N = 10$ Mio. How should we choose K in order to the forward-swap has value 0 at time $t = 0$.

Exercise 8

Let us consider a Receiver-Swap with the fixed interest rate $K = 4.8\%$, the face value $N = 10$ Mio. EUR, maturity 5 years and annual payment. The continuous spot rate curve today at time $t = 0$ is given by

T	1	2	3	4	5	6	7	8
R(0,T)	4,62%	4,65%	4,70%	4,75%	4,79%	4,82 %,	4,82%,	4,82%

a) Calculate the forward interest rate $F(0, T_i, T_{i+1})$.

b) Calculate the simple spot-rates $P(0, T_i)$.

c) Calculate the value of swaps and the "faire" swap rate $S_{0,5}(0)$.

d) Calculate the value of 6x2-swaps (i.e. the forward-swaps with $T_0 = 6, T_1 = 7$ and $T_2 = 8$) with $K = 5\%$, $N = 10$ Mio. EUR. How should we choose K in order to the forward-swap has value 0 at time $t = 0$.