On quasi-infinitely divisible distributions.

Alexander Lindner (Ulm University)

Abstract:
A quasi-infinitely divisible distribution on the real line is a probability distribution whose characteristic function allows a Lévy-Khintchine type representation with a signed Lévy measure, rather than a Lévy measure. Quasi-infinitely divisible distributions appear naturally in the factorization of infinitely divisible distributions. Namely, a distribution $\mu$ is quasi-infinitely divisible if and only if there are two infinitely divisible distributions $\mu_1$ and $\mu_2$ such that $\mu_1 \ast \mu = \mu_2$. We study properties of quasi-infinitely divisible distributions in terms of their characteristic triplet, such as properties of supports, finiteness of moments, continuity properties and weak convergence, with various examples constructed. In particular, it is shown that the set of quasi-infinitely divisible distributions is dense in the set of all probability distributions with respect to weak convergence. Further, it is proved that a distribution concentrated on the integers is quasi-infinitely divisible if and only if its characteristic function does not have zeroes, with the use of the Wiener-Lévy theorem on absolutely convergent Fourier series. A similar characterisation is not true for non-lattice probability distributions on the line. The talk is based on joint work with Lei Pan (Ulm) and Ken-iti Sato (Nagoya).