Nonparametric estimation of the characteristics of stationary Lévy random fields

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Consider a stationary real-valued infinitely divisible random field $X = \{X(t), t \in \mathbb{R}^d\}$ with spectral representation

$$X(t) = \int_{\mathbb{R}^d} f_t(x) M(dx), \quad t \in \mathbb{R}^d,$$

where f_t is a kernel function which is integrable in a proper sense and M is an infinitely divisible independently scattered random measure. We propose a method for the estimation of the characteristic Lévy triplet of M (shift and scale parameters as well as Lévy density) out of the observations of X. For that, f_t is assumed to be approximated by a piecewise constant function which can be estimated from the data together with the characteristic Lévy triplet of X. The consistency of the method is shown.

If X has finite second moments then parametric minimum contrast and non–parametric Fourier–based methods can be applied to estimate f_t via the covariance function of X. What happens if X is not square integrable? We propose a non–parametric estimator for f_t in the case where X is an α-stable moving average with $\alpha \in (1, 2]$, i.e., $f_t(x) = f(t - x)$ for all $t, x \in \mathbb{R}^d$ and M is an α-stable random measure with Lebesgue control measure. For that, we assume f to be a symmetric kernel function with compact support. We prove consistency of the estimator and apply the method to a numerical example.