A terminal wealth problem in an illiquid market under a drawdown constraint

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In the classical theory of optimal investments on a finite horizon, it is assumed that the underlying financial market is perfectly liquid. However, this is not always a realistic assumption. Therefore, we consider an optimal investment problem, where observing and trading is only possible at discrete random times, which converge to the finite horizon. Additionally, we introduce a drawdown constraint, which requires that the investor's wealth does not fall under a prior fixed percentage of its running maximum. This financial market is completed by considering two financial assets, which are driven by inhomogeneous Lévy processes. Moreover a general utility function is assumed. In this setting we solve the investment problem using a related limsup Markov Decision Process. Making use of that connection we are able to show that there exists an optimal portfolio and that the value function can be characterized as the unique fixed point of the maximal reward operator, which is uniformly integrable and satisfies a terminal condition. Furthermore, we verify the existence of an optimal stationary policy, which is given by a maximizer of the value function.

Finally, under some mild assumptions, the value function can be approximated by the value function of a contracting Markov Decision Process. Hence, we are able to use Howard's policy improvement algorithm to approximate the value function as well as an optimal policy. These results are illustrated in a numerical example.