Statistical Methods for Stochastic Differential Equations

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Contents

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Preface

The chapters of this volume represent the revised versions of the main papers given at the seventh Séminaire Européen de Statistique on “Statistics for Stochastic Differential Equations Models”, held at La Manga del Mar Menor, Cartagena, Spain, May 7th-12th, 2007. The aim of the Séminaire Européen de Statistique is to provide talented young researchers with an opportunity to get quickly to the forefront of knowledge and research in areas of statistical science which are of major current interest. As a consequence, this volume is tutorial, following the tradition of the books based on the previous seminars in the series entitled:

- Networks and Chaos – Statistical and Probabilistic Aspects.
- Time Series Models in Econometrics, Finance and Other Fields.
- Stochastic Geometry: Likelihood and Computation.
- Complex Stochastic Systems.
- Extreme Values in Finance, Telecommunications and the Environment.
- Statistics of Spatio-temporal Systems.

About 40 young scientists from 15 different nationalities mainly from European countries participated. More than half presented their recent work in short communications; an additional poster session was organized, all contributions being of high quality.

The importance of stochastic differential equations as the modeling basis for phenomena ranging from finance to neurosciences has increased dramatically in recent years. Effective and well behaved statistical methods for these models are therefore of great interest. However the mathematical complexity of the involved objects raise theoretical but also computational challenges. The Séminaire and the present book present recent developments that address, on one hand, properties of the statistical structure of the corresponding models and, on the other hand, relevant implementation issues, thus providing a valuable and updated overview of the field.

The first chapter of the book, written by Michael Sørensen, describes the application of estimating functions to diffusion type models. Estimating functions are a comparatively recent tool to estimate parameters of discretely observed stochastic processes. They generalize the method of maximum likelihood estimation by searching for the roots of a so-called estimating equation, but have the advantage that these equations can typically be calculated and solved more easily than the likelihood equations, which often require extensive calculations. The idea is to approximate the likelihood equations, and in certain situations estimating functions provide fully efficient estimators. Maximum likelihood estimations is discussed as a particular case.

The second chapter, written by Per Mykland and Lan Zhang addresses the modeling of high frequency data of financial prices. The considered model is assumed to be a semimartingale plus an additive error, the so called microstructure noise. This noise causes difficulty in estimation, since its impact on the estimators may be higher than that of the relevant model parameters. An approach is presented to overcome these difficulties, using multiscale realized volatility. It is in particular shown how statistical data can be used in a sensible way for the trading of options, hence combining the probabilistic part of mathematical finance with statistical issues that may arise by misspecification of the model or other errors.

In Chapter 3, Jean Jacod treats inference for general jump diffusion processes based on high frequency data. This means that one observes a stochastic process at equidistant time points between time 0 and time T, where the corresponding interval between two consecutive observation times is small and in the limit tends to zero. Such models have many applications, in particular in finance, where one is interested in estimating the integrated volatility. A number of estimation techniques for such general processes are presented, mainly based on variants of the quadratic variation, and the corresponding limit theorems are explained. This allows in particular to develop tests to distinguish whether processes have jumps or not.

Chapter 4, written by Omiros Papaspiliopoulos and Gareth Roberts, focuses on computational methods for the implementation of likelihood based inference procedures for diffusion models. After a detailed overview on various simulation techniques for diffusions, the exact simulation method is presented with particular emphasis on the simulation of conditioned diffusions. Rather than using an Euler approximation scheme, these simulation methods simulate the path of a (conditioned) diffusion exactly, without any discretization error. The exact simulation method can then be combined with Monte Carlo techniques to compute efficiently maximum likelihood and Bayesian estimators for diffusions.

The short chapter 5, written by Fabienne Comte, Valentine Genon-Catalot and Yves Rozenholc, gives insight on non parametric methods for stochastic differential equations models, several methods being presented and the corresponding convergence rates investigated. Several examples are used to illustrate the behaviour of the suggested procedures.

In the short Chapter 6, Peter Brockwell and Alexander Lindner discuss some recent stochastic volatility models where the driving process is a Lévy process with jumps. After presenting the motivations for such models and their properties, estimation methods are described.

Finally in the short Chapter 7, written by Grigorios Pavliotis, Yvo Pokern and Andrew Stuart, the modeling of the multiscale characteristic that may be present in data is addressed and the procedures that can be used to find a useful diffusion approximation are described. Some examples from physics and molecular dynamics are presented.

The Séminaire Européen de Statistique is an activity of the European Regional Committee of the Bernoulli Society for Mathematical Statistics and Probability. The scientific organization for the seventh Séminaire Européen was in the hands of Ole E. Barndorff- Nielsen, Aarhus University, Bärbel Finkenstädt, University of Warwick, Leonhard Held, University of Zürich, Ingrid van Keilegom, Université Catholique de Louvain, Alexander Lindner, Technical Braunschweig University, Enno Mammen, University of Mannheim, Gesine Reinert, University of Oxford, Michael Sørensen, University of Copenhagen and Aad van der Vaart, University of Amsterdam. The seventh Séminaire was part of the second series of the European Mathematical Society Summer Schools and Conferences in pure and applied Mathematics and, as such, was funded by the European Commission as a Marie Curie Conference and Training Course under EU Contract MSCF-CT-2005-029473. The organization committee is grateful to the Technical University of Cartagena and in particular to its Department of Applied Mathematics and Statistics, for providing support to the Conference.