Data-Efficient Convolution on Irregular FE-Meshes Applied to a 3D Fracture Healing Simulation

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The present work deals with a specific step of a fracture healing simulation (see e.g. [1], [2]). In this step, the given concentration distribution of a biological tissue, such as bone, is *smoothed* (i.e. *smeared* over the three-dimensional healing domain). The current implementation of this *smoothing* operation in the form of a matrix-vector multiplication turns out to be a runtime-critical component in *large* simulation scenarios.

We therefore transfer the original problem definition step by step into an alternative problem definition. Ultimately, this leads to an algorithm in which the smoothing is performed in multiple steps, i.e. iteratively. In each of these steps, a matrix vector product is evaluated. The advantage of this approach over the original approach is that the matrix used here requires significantly less memory space. An analytical estimation indicates that a considerable increase in performance can be expected.

Extensive numerical experiments reveal some characteristics of the different methods and confirm theoretical considerations. Here we focus especially on the numerical errors and the runtime of the algorithms. By using the iterative approach, acceleration factors in the order of 70 can be observed, which is primarily due to the reduction of the memory consumption for the matrix.

Final critical considerations show in particular the influence of the row-wise normalization of the matrix on the result of the smoothing. In addition, some disadvantages of the currently used matrix class with respect to the flexibility in the choice of data types are presented, which suggest a modification of our implementation. This would lead to a further drastic reduction in memory consumption and thus to an additional increase in performance.

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References

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