Preconditioning the Cahn-Hilliard Equation

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We consider electrode particles of lithium ion batteries during the discharge process. The separation into lithium poor and lithium rich phases due to changes in the host material can be described with a phase-field model, resulting in the fourth order Cahn-Hilliard equation [1].

Difficulties solving this initial boundary value problem are on the one hand the highly nonlinear character of the equations and on the other hand the almost sharp moving phase boundary and the different time scales on which the separation process take place. In particular for three-dimensional electrode particles we need a robust solver overcoming these problems.

For the spatial discretization we employ a finite element method. The semi-discrete problem is linearized with Newton's method and for the time stepping we use an adaptive second order scheme [2]. For the fast solution of the large linear systems arising in each Newton step we use the matrix-free framework within the open-source finite element library deal.II [3, 4]. Based on the work [5] we developed a preconditioner, which is applicable also in the matrix-free framework.

We have implemented and tested an efficient finite element solver for the Cahn-Hilliard equation. The matrix-free solver is fast and does not need to store any matrices, in particular it can be run in parallel using MPI. Analyzing our developed preconditioner, the number of GMRES steps show only a moderate dependence on the time step and the mesh size. A comparison to a standard Jacobi preconditioner is shown.

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