Optimizing MGRIT and Parareal coarse-grid operators for linear advection

Hans De Sterck[a], Robert D. Falgout[b], Stephanie Friedhoff[c], Oliver A. Krzysik[d], Scott P. MacLachlan[e]*

a) University of Waterloo, www.uwaterloo.ca, Canada
b) Lawrence Livermore National Laboratory, www.llnl.gov, USA
c) Bergische Universität Wuppertal, www.uni-wuppertal.de, Germany
d) Monash University, www.monash.edu, Australia
e) Memorial University of Newfoundland, www.mun.ca, Canada
*) friedhoff@math.uni-wuppertal.de

Hardware trends and scaling limits have driven the development of algorithms that allow space-time parallelism. These methods consider the solution of time-dependent systems of partial differential equations (PDEs), and allow simultaneous solution across multiple time steps, in contrast to classical time-stepping approaches considering the sequential solution of one time step after the other. While parallel-in-time methods, such as multigrid-reduction-in-time (MGRIT) [1] and Parareal [2] have been very successful for parabolic equations, it has often been observed that their performance suffers dramatically when applied to advection-dominated problems or purely hyperbolic PDEs using standard rediscretization approaches on coarse grids.

In this talk, we apply MGRIT or Parareal to the coonstant-coefficient linear advection equation, appealing to existing convergence theory to provide insight into the typically non-scalable or even divergent behavior of these solvers for this problem. To overcome these failings, we replace rediscretization on coarse grids with near-optimal coarse-grid operators that are computed by applying optimization techniques to approximately minimize error estimates from the convergence theory.

Our approach is tested on discretizations of various orders that use explicit or implicit Runge-Kutta time integration with upwind-finite-difference spatial discretizations, for which we obtain fast and scalable solvers in all cases. Parallel tests also demonstrate significant speed-ups over sequential time-stepping. Our main finding is that, in order to obtain fast convergence as for parabolic problems, coarse-grid operators should take into account the behavior of the hyperbolic problem by tracking the characteristic curves. Our insight is implemented for linear advection using an optimization approach, but the principle is general, and provides a key idea for solving the long-standing problem of efficient parallel-in-time integration for hyperbolic PDEs.

References

[1] R. Falgout et al.: *Parallel time integration with multigrid.* SISC, 2014.
[2] J.-L. Lions et al.: *Résolution d'EDP par un schéma en temps pararéel.* C. R. Acad. Sci., 2001.