Numerical Algorithms for Forward-Backward Parabolic PDEs

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Partial differential equations (PDEs) of forward-backward parabolic type have been advocated for image enhancement since more than five decades [1,2]. The ill-posedness of the backward parabolic term, however, makes it challenging to design adequate numerical algorithms. Further difficulties can arise from pronounced anisotropies and specific application-driven requirements such as stability in the maximum norm or a high degree of rotation invariance.

In this talk we discuss a number of fairly general numerical ideas to handle these challenges [3,4]. This includes nonstandard finite difference approximations and sequential splittings into highly localised processes, but also ideas from the numerics of hyperbolic PDEs such as upwinding, minmod schemes, and curvature limiters. Combining these concepts in an appropriate way allows to come upwith stability guarantees that reflect key properties of the continuous models.

Our experiments with forward-and-backward (FAB) diffusion and evolutions of Gabor type demonstrate the practical usefulness and the good performance of the resulting algorithms.

Joint work with Martin Welk (UMIT, Hall, Austria) and Guy Gilboa (Technion, Haifa, Israel).

References:

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