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Numerik von gewöhnlichen Differenzialgleichungen
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Sheet 5

Due May 19, 2016.

Exercise 1 (Difference Equation I)

(a) Compute the general solution of the following difference equation

$$y_{k+3} - 4y_{k+2} + 5y_{k+1} - 2y_k = 0, \qquad k = 0, 1, \ldots .$$

(b) Solve the following difference equations

(i) $y_{k+2} - 2y_{k+1} - 3y_k = 0, \qquad y_0 = 0, y_1 = 1.$

(ii) $y_{k+1} - y_k = 2^k, \qquad y_0 = 0.$

Exercise 2 (Difference Equation II)

Consider the following linear homogene difference equation with real (or complex) coefficients α_k

$$\sum_{k=0}^{s} \alpha_k w_{j+k} = 0, \qquad j = 0, 1, \ldots, \quad \alpha_s \neq 0.$$

(a) If z_1 is a root of the characteristic polynomial $\rho(z) := \sum_{k=0}^{s} \alpha_k z^k$, then $w_j := z_1^j$ is a solution of the difference equation.

(b) If z_2 is a double zero spot of $\rho(z)$, then $w_j := z_1^j$ and $v_j := j z_1^j$ both solve the difference equation.

(c) If $z_1, \ldots, z_s$ are pairwise different single zero spots of $\rho(z)$, then the general solution of the difference equation is given by $\sum_{k=1}^{s} c_k z_k^j$, where $c_k \in \mathbb{R}$.

Exercise 3 (Order of Consistency)

(a) Compute the exact order of consistency for the following multistep method

$$y_{k+2} - y_k = \frac{h}{3} \left[f(t_{k+2}, y_{k+2}) + 4f(t_{k+1}, y_{k+1}) + f(t_k, y_k) \right].$$

(b) Compute the order consistency in dependence of $\gamma \in \mathbb{R}$ for the following multistep method

$$y_{k+2} + \gamma(y_{k+2} - y_{k+1}) - y_k = h \frac{3+\gamma}{2} \left[f(t_{k+2}, y_{k+2}) + f(t_{k+1}, y_{k+1}) \right].$$

For which γ is the method zero-stable?

Exercise 4 (Construction of multistep methods)

Determine the coefficients $b_0, b_1, b_2 \in \mathbb{R}$ of the two-step Adams-Moulton-method

$$y_{k+2} = y_{k+1} + h \sum_{j=0}^{2} b_j f(t_{k+j}, y_{k+j})$$

(i) with the corresponding interpolation method,

(ii) with the help of the linear equation system for determination of the order of consistency for linear multistep methods.