Prof. Dr. Karsten Urban
M. Sc. Mazen Ali
Institute for Numerical Mathematics
Ulm University

Seminar Reduced Basis Methods
SoSe 2017

# Seminar Outline

February, 2017

## Guidelines and Grading

In the following you will find a list of possible topics that can be chosen for this seminar. The variety of topics ranges from purely theoretical to topics focused on numerical implementation. In this seminar your task is to research the chosen topic, obtain an overview and a good understanding of the subject area and present it. The core of this seminar is based on an introductory book about *Reduced Basis Methods* (see references in the end) with a comprehensive overview. This book serves as a good starting point for your project.

At the end of this outline we provide a list of references and links. However, you are by no means limited to this list and should research the topic beyond if necessary, e.g., by searching relevant references in the provided material.

Together with a list of topics we provide some guiding questions and remarks that should give you an idea of how to approach the subject. Of course, these questions serve as a basic guideline and do not limit you in the variety of problems you can address. For numerical experiments you should take a look at some of the referenced works to get an idea of which problems you can experiment with and how to perform and present numerical results.

You have to prepare an approximately 5 page report where you give an overview over your topic and present the work you did. You also have to prepare a 30 minute talk (+ 15 minutes for discussion) and present it in the seminar.

The presentations will start some time around May-June, on a weekly basis, with 1-2 presentations per block. Details will depend on the number of participants and will be announced before the beginning of the semester. The report is typically due 1 week prior to the presentations. If applicable, you also have to submit your code together with the report. Your report (+ code) and presentation will be the basis of your evaluation.

Should you have any questions, send an email to mazen.ali@uni-ulm.de. We will meet in the beginning of the semester to discuss any open questions. Dates and details will be announced on the institutes web page.

## Registration

The registration is open as of now. The seminar slots and topics will be distributed on a first come first serve basis. To register send an email to mazen.ali@uni-ulm.de. In the email state your **full name, student number, study program and 5 choices for a topic** in descending order of preference.

## List of Topics

The list of topics is tentative and may be extended depending on the number of participants.

1. Model reduction.

   - What is *model (order) reduction*?
   - What kind of problems and applications are considered?
   - What methods are there for MOR?

- What are the general principles common to all (or most) methods?
- This topic is a general overview topic. You should familiarize yourself with what MOR is for, what it includes and have a basic understanding of the methods.

2. Theoretical foundation of RB.
   - What is the theoretical foundation for reduced basis methods (RB or RBM)?
   - When is the use of RB justified from a theoretical point of view?
   - How can you interpret RB (geometrically, algebraically)?
3. Linear affine problems.
   - How do you numerically compute RB for a linear and affine problem?
   - Perform and present numerical experiments for the Stokes problem.
4. Nonaffine problems.
   - Why is affineness an important property for RB?
   - How can you treat nonaffine problems?
   - Perform and present numerical experiments with a nonaffine problem(s).
5. Nonlinear problems.
   - What difficulties in connection with RB appear when dealing with nonlinear problems?
   - How can you treat nonlinear problems?
   - Perform and present numerical experiments with a nonlinear problem(s).
6. Optimization.
   - How can RB be used in conjunction with optimization problems?
   - Perform and present numerical experiments with a parametrized optimal control problem(s).
7. Discretization methods.
   - What type of discretization methods can be used with RB?
   - What are the differences in theory and practice between the different types of discretizations?
   - What type of applications favor which kind of discretizations?
   - Perform and present numerical experiments with different types of discretizations.
8. Geometry parametrization.
   - What is GP and what kind of problems does it apply to?
   - Perform and present numerical experiments with a problem(s) requiring GP.
9. Adaptive grids.
   - What is adaptive grid generation?
   - How does it affect RB?
   - For what kind of problems are adaptive grids meaningful?
   - Perform and present numerical experiments with a problem(s) requiring an adaptive grid.
10. Maxwell equation.
    - What is the Maxwell equation and where is it applied?
    - How can you apply RB to the Maxwell equation?
    - Perform and present numerical experiments with the Maxwell equation using RB.

11. Eigenvalue problems.
    - What is an eigenvalue problem and what are the applications?
    - How can you apply RB to an eigenvalue problem?
    - Perform and present numerical experiments with an elliptic eigenvalue problem using RB.
12. *hp*-RB
    - What is the *hp*-RB method?
    - When is it applied?
    - Perform and present numerical experiments using the *hp*-RB method.

## Material

For software we recommend using MATLAB with the software package RBmatlab. In the following is a list of links and references that you can use to start with your project. Of course, not all references are relevant for your topic.

Alfio Quarteroni, Andrea Manzoni, and Federico Negri. *Reduced basis methods for partial differential equations*, volume 92 of *Unitext*. Springer, Cham, 2016. ISBN 978-3-319-15430-5; 978-3-319-15431-2. doi: 10.1007/978-3-319-15431-2. URL `http://dx.doi.org/10.1007/978-3-319-15431-2`. An introduction, La Matematica per il 3+2

This book is an introduction to RB and is a good starting point for all topics. We will provide a link to an ebook version temporarily available for the purpose of this seminar.

`http://www.ians.uni-stuttgart.de/MoRePaS`

This is the web page of the model order reduction community It contains links to papers, latest news and software packages (including RBmatlab). It also provides a brief overview over the research areas in MOR.

Mazen Ali, Kristina Steih, and Karsten Urban. Reduced basis methods with adaptive snapshot computations. *Advances in Computational Mathematics*, pages 1–38, 2016. ISSN 1572-9044. doi: 10.1007/s10444-016-9485-9. URL `http://dx.doi.org/10.1007/s10444-016-9485-9`

MASAYUKI Yano. A reduced basis method for coercive equations with an exact solution certificate and spatio-parameter adaptivity: Energy-norm and output error bounds. *Preprint, Univ. of Toronto, submitted*, 2016

Sebastian Ullmann, Marko Rotkvic, and Jens Lang. POD-Galerkin reduced-order modeling with adaptive finite element snapshots. *J. Comput. Phys.*, 325:244–258, 2016. ISSN 0021-9991. doi: 10.1016/j.jcp.2016.08.018. URL `http://dx.doi.org/10.1016/j.jcp.2016.08.018`

Martin Drohmann, Bernard Haasdonk, and Mario Ohlberger. Reduced basis method for finite volume approximation of evolution equations on parametrized geometries. In *Proceedings of ALGORITMY*, volume 2008, pages 111–120, 2009

Fumagalli, Ivan, Manzoni, Andrea, Parolini, Nicola, and Verani, Marco. Reduced basis approximation and a posteriori error estimates for parametrized elliptic eigenvalue problems. *ESAIM: M2AN*, 50(6):1857–1885, 2016. doi: 10.1051/m2an/2016009. URL `http://dx.doi.org/10.1051/m2an/2016009`

Jens L. Eftang, Anthony T. Patera, and Einar M. Rø nquist. An "*hp*" certified reduced basis method for parametrized elliptic partial differential equations. *SIAM J. Sci. Comput.*, 32(6):3170–3200, 2010. ISSN 1064-8275. doi: 10.1137/090780122. URL `http://dx.doi.org/10.1137/090780122`

Yanlai Chen, Jan S. Hesthaven, Yvon Maday, and Jerónimo Rodrí guez. Certified reduced basis methods and output bounds for the harmonic Maxwell's equations. *SIAM J. Sci. Comput.*, 32(2):970–996, 2010. ISSN 1064-8275. doi: 10.1137/09075250X. URL `http://dx.doi.org/10.1137/09075250X`

These are some papers that may be relevant for your topic.