Intraday Renewable Electricity Trading: Advanced Modeling and Optimal Control

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Abstract This paper is concerned with a new mathematical model for intraday electricity trading involving both renewable and conventional generation. The model allows us to incorporate market data e.g. for half-spread and immediate price impact. The optimal trading and generation strategy of an agent is derived as the viscosity solution of a second-order Hamilton-Jacobi-Bellman (HJB) equation for which no closed-form solution can be given. We thus construct a numerical approximation allowing us to use continuous input data. Numerical results for a portfolio consisting of three conventional units and wind power are provided.

1 Introduction

Due to the extensive rise of renewable power supply as a response to the global climate change, electricity short-term markets like EPEX SPOT, in particular continuous intraday trading, gained more importance. This, in turn, motivates the interest in mathematical modeling of such trading as a basis for deeper understanding and optimization. Early work in that direction can be found in [4]. In [1], the authors derive a Hamilton-Jacobi-Bellman (HJB) equation for determining an optimal trading strategy by modeling the dynamics of the electricity market by *stochastic differential equations* (SDEs) and formulating a corresponding *value function* to be

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optimized. The specific market model in [1] allows to solve the arising HJB analytically, i.e., the authors derive a solution formula. The starting point of this paper is a statistical analysis of EPEX SPOT data, which shows that some of the model assumptions in [1] are not satisfied under real market conditions. Thus, we introduce a more sophisticated model. The arising HJB equation can no longer be solved analytically; the value function is shown to be the unique *viscosity solution* of this HJB equation. Thus, we need an appropriate numerical scheme.

From an economical point of view, the main new ingredients of our model are: (1) Portfolio of renewable and conventional energy represented by a cost function that reflects the stepwise merit order of a portfolio rather than a systemwide quadratic function; (2) Pricing model using time-varying half-spread and being capable of representing time-varying liquidity; (3) Approximation of market data for half-spread and instantaneous price impact; (4) Variable penalty depending on the state of the market at final time. The main focus of this paper is a novel application-related modeling of the intraday trading and the determination of a numerical approximation for this problem. We show an example of a real-world problem and compute the optimal trading strategy. The remainder of this paper is as follows: In §2, we introduce the new model and the arising HJB equation, §3 is devoted to the presentation of numerical experiments and we finish by an outlook.

2 A New Mathematical Model

In order to take both renewable and conventional generation into account, our model is based upon the consideration of an agent owning both kinds of power plants and aiming at selling a combination of renewable[1] and additionally conventionally produced electricity. In detail, depending on the weather forecast and the expected price at the final time, a combination of conventional and renewable electricity is sold at the day-ahead market. With this sold amount, the agent starts the continuous intraday trading aiming at maximizing her profit by determining an optimal trading strategy as well as an optimal production of conventional power[2]. We are now going to describe both involved frameworks, namely the trading model including day-ahead as well as intraday trading and the stochastic model of the dynamics.

Day-Ahead and Intraday Electricity Trading: Consider a delivery hour h on day d. The day before, the *day-ahead auction* takes place with gate closure at 12pm. In this auction, each participant can offer (ask) or request (bid) a certain demand of electricity at a specific price. Then, a clearing price is set and power is exchanged accordingly. Next, the *continuous intraday trading* starts at 3pm on day d-1 and closes half an hour before the actual delivery hour h, see Figure 1.

Dynamics of the Electricity Market: The dynamics of the market includes the forecasted renewable power production and the price process. The latter one is in-

[1] In our numerical experiments, we consider wind energy.

[2] That means that we do not optimize day-ahead and intraday trading at the same time.

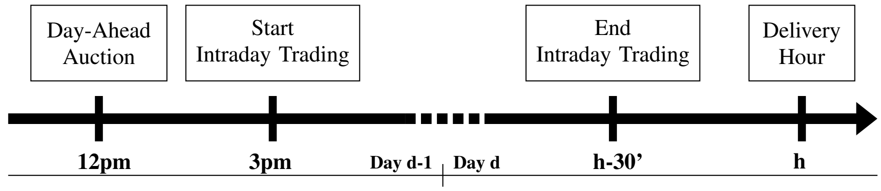

Fig. 1 Scheme of continuous intraday trading.

fluenced by the current trading activity of the agent. We use stochastic processes and derive stochastic differential equations (SDEs).

Forecast model for the renewable power. By $D = (D_t)_{0\leq t\leq T}$ we denote the *forecasted production* of renewable electricity during the trading session. The uncertainty is modeled by means of the dynamics $dD_t = \mu_D\,dt + \sigma_D\,dW_{t,D}$, where μ_D is the drift, σ_D is the volatility and $(W_{t,D})_{0\leq t\leq T}$ is a standard Brownian motion. For the sake of simplicity, this variable is unbounded, whereas in the real world, there are restrictions by zero (no wind) and the maximum capacity of the wind farm.

Agent's position. The *financial position* resulting from the agent's trading activity is denoted by $X = (X_t)_{0\leq t\leq T}$. The agent participates in the intraday market with continuous trading at rate $q_t \in \mathcal{Q} \subset \mathbb{R}$ ($q_t > 0$ means buying, $q_t < 0$ selling), i.e., $dX_t = q_t\,dt$ and we denote by $X^{q,t,x}$ the solution of the SDE starting from x at t (for $t = 0$, x_0 is the amount of electricity sold on the day-ahead market).

Price model. The *execution price* is the price the agent pays (receives) when actually buying (selling). We require a more advanced approach of the pricing model as in [1], where the half-spread and its time variability as well as the time variability of the immediate price impact are ignored. Incorporating these effects, the execution price depends on a number of quantities to be introduced now. First, we denote by $Y = (Y_t)_{0\leq t\leq T}$ the sum of the *mid price* of energy and the *permanent impact* of the agent's trading modeled by some function $\psi : \mathbb{R} \to \mathbb{R}$. Its dynamics is modeled by the SDE $dY_t = (\mu_Y + \psi(q_t))dt + \sigma_Y\,dW_{t,Y}$, where μ_Y is the drift, σ_Y is the volatility and $(W_{t,Y})_{0\leq t\leq T}$ is a standard Brownian motion. We denote by $Y^{t,y}$ the solution of the SDE starting from y at t. The next ingredient is the *half-spread* $h : [0,T] \to \mathbb{R}$, i.e., the half of the distance between the best ask and the best bid price. This is data which can be retrieved from the market. With all these quantities at hand, the execution price $P^{q,t,y} = (P^{q,t,y}_s)_{0\leq s\leq T}$ is modeled as

$$P^{q,t,y}_s := Y^{t,y}_s + \frac{|q_s|}{q_s} h(s) + \varphi(t, q_s), \tag{1}$$

i.e., the permanently impacted mid price plus (minus) the half-spread and the instantaneous price impact $\varphi : [0,T] \times \mathcal{Q} \to \mathbb{R}$.

Conventional production/payoff. At the end of the trading session T, the agent chooses how much electricity $\xi \in \mathbb{R}^+_0$ she will produce during the delivery period. In doing so, she also has the option to place a final buy or sell market order, potentially resulting in $\xi \neq -Z_T$, with $Z_t := X_t + D_t$ being the sum of the forecasted production from renewables and what has been sold by the agent so far. For example, she could further increase her sell position and production. The final market order goes along with costs due crossing the half-spread $h(T)$ and potentially executing limit orders whose prices are worse than the best bid/ask price due to a penalty $\alpha : \mathbb{R} \to \mathbb{R}_0^+$. The arising cost per unit depends on the state of the market at T. The terminal payoff is

$$g(\xi, Y_T, Z_T) := -c(\xi) + (\xi + Z_T)\left(Y_T - (h(T) + \alpha)\frac{|\xi + Z_T|}{\xi + Z_T}\right), \tag{2}$$

where $c : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ models the cost of the conventional generation.

Value function. The value function corresponds to the agent's cash, so that an optimal strategy yields maximal cash. The total running profit from the continuous trading in the intraday market is given by $f^q(s;t,y) := -q_s P_s^{q,t,y}$. Denoting by $Z^{q,t,z}$ the solution of the SDE $dZ_t = dD_t + dX_t = (q_t + \mu_D)\,dt + \sigma_D\,dW_{t,D}$ starting from z at t, the resulting value function $V : [0,T] \times \mathcal{U} \to \mathbb{R}$ reads

$$V(t,y,z) := \sup_{(q,\xi) \in \mathcal{Q} \times \mathbb{R}} \mathbb{E}\left[\int_t^T f^q(s;t,y)\,ds + g(\xi, Y_T^{t,y}, Z_T^{q,t,z})\right], \tag{3}$$

where $\mathcal{U} := \mathcal{Y} \times \mathcal{Z} \subset \mathbb{R}^2$ is a rectangle (in order to ensure well-posedness of the optimization in (3), [3]). We prescribe Dirichlet conditions on the boundary $\partial\mathcal{U}$.

Hamilton-Jacobi-Bellman (HJB) Equation: Following the well-known *dynamic programming principle* (e.g. [6, Ch. 4]), we derive the HJB equation: Find $W : [0,T] \times \mathcal{U} \to \mathbb{R}$, $W = W(t,y,z)$, such that

$$\begin{aligned} &\partial_t W + \mu_Y \partial_y W + \mu_D \partial_z W + \frac{1}{2}\sigma_Y^2 \partial_{yy} W + \frac{1}{2}\sigma_D^2 \partial_{zz} W \\ &\quad + \sup_{q(t) \in \mathcal{Q}} \left\{ -\left(y + h(t)\frac{|q(t)|}{q(t)} + \varphi(t, q(t))\right) q(t) + q(t)\,\partial_z W + \psi(q(t))\,\partial_y W \right\} = 0, \end{aligned} \tag{4}$$

for $(t,y,z) \in [0,T) \times \mathcal{U}$ with terminal condition $W(T,y,z) = g(T,y,z)$, $(y,z) \in \mathcal{U}$. One can show that this problem is well-posed and that the unique *viscosity solution* W is the value function V in (3). Due to the form of (4), we cannot expect a first-order condition for the control $q(t)$ and we have to resort to numerical solvers.

3 Numerical Experiment

Finally, we report on results of a numerical experiment concerning (4) using the following data: $\mathcal{U} := [-50, 250] \times [-1645, 145] \subset \mathbb{R}^2$ and $T = 17.5h$. We use a finite difference discretization from [5] with 56×301 points in space and 100 points in time. In particular, central differences are used for the approximation of the first-order terms with additional artificial diffusion, which results in a stable, consistent and monotone scheme converging to the viscosity solution, [5]. We use the well-known *policy iteration* in every time-step and the control is maximized over a discrete set (as no first-order conditions are available). Finally, the optimal conventional generation is computed as the maximum value of (2) w.r.t. ξ using Matlab's intlinprog with the interior point method.

Boundary conditions: Similar to option pricing, the choice of appropriate boundary conditions (here for y and z) is delicate. Here, we use a similar but easier HJB allowing for a closed-form solution on some $\mathcal{U} \subset \mathbb{R}^2$. Then, we prescribe the boundary values of this function as Dirichlet conditions on $\partial\mathcal{U}$.

Data: We use the data $\mu_D := \mu_Y := 0.0$, $\sigma_D := \sigma_Y := 0.1$. The functions $\varphi(\cdot,\cdot)$ and $h(\cdot)$ are least-squares 5th order polynomial approximations of market data from Q2/2015 ($\psi(t) = 0$). The penalty is given by market data as $\alpha(x) := 0.5 \cdot (|x| - 20)\chi_{20<|x|\leq 45} + ((|x|-45)+12.5)\chi_{45<|x|\leq 145}$. We consider three conventional units, namely a hard coal plant with 25 €/MWh variable cost and min-max capacity of 250-500 MW, one combined cycle gas turbine (CCGT) unit (35 €/MWh, 100-400 MW) and open cycle gas turbine (OCGT) unit (60 €/MWh, 60-600 MW).

Our results for the **optimal conventional generation** ξ are displayed in Figure 2. Let us comment on the case where $Z_T = -500$ MWh. As long as the final mid price is below 25€/MWh, the agents buys the maximal amount of 145 MWh (recall, that $y \in [-1645, 145]$) and uses the power plant with the lowest marginal costs (hard coal) accordingly, i.e. the remaining 355 MWh. Once the final mid price is 25 to 35€/MWh (i.e., above the marginal cost of hard coal, but below the marginal cost of CCGT) it is optimal to produce at maximum capacity with the cheapest conventional power plant (i.e. 500 MWh by hard coal) and no final market order is required. If the final mid price exceeds 35€/MWh, the agent sells as much electricity as possible (145 MWh) and produces exactly that amount with the CCGT plant at 35€/MWh, which is possible because its capacity is 100-400 MW. Finally, no matter how high the final mid price is, the OCGT unit with the highest marginal cost is not used, since there is not enough sell volume on the market. These results are clearly reasonable.

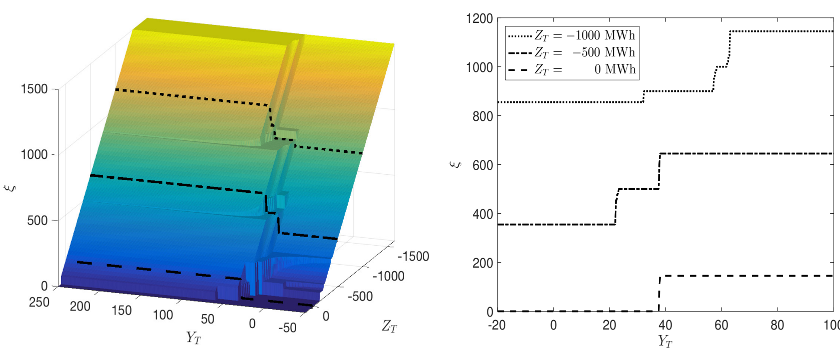

Fig. 2 Optimal conventional generation ξ as a function of Y_T and Z_T (left) as well as for some values of Z_T (right; the lines correspond to those on the left graph).

Trading rate: Figure 3 shows the optimal trading rate over the trading window $t \in [0\text{h}, 17.5\text{h}]$. In both cases, we fix $Z_t \equiv$-499.4 MWh (the non-integer numbers arise from the discretization w.r.t. y and z). For the mid price, we choose $Y_t \equiv$59.25 €/MWh (left) and $Y_t \equiv$13.98 €/MWh (right). In the left plot, the trading rate is negative (selling), which is reasonable since $Z_t \equiv$-499.4 MWh means that the agent has only marketed the cheapest power plant and $Y_t \equiv 59.25$ €/MWh means that the execution price is above the marginal costs of the second cheapest power plant. Note, that the absolute value of the trading rate substantially increases around 15h, since half-spread and immediate price impact are minimal there. In the right plot, the execution price is below the marginal costs of the cheapest power plant, the agent buys electricity and reduces the production of the marketed power plant.

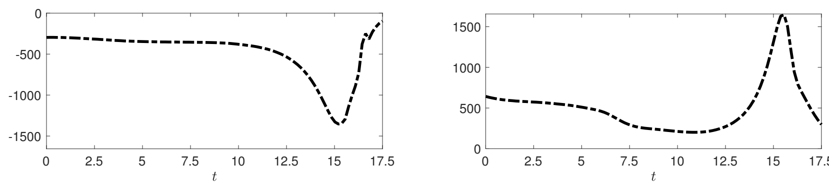

Fig. 3 Optimal trading rate over the trading window $t \in [0, 17.5]$ for $Z_t \equiv$-499.4 MWh and $Y_t \equiv$59.25 €/MWh (left) as well as $Y_t \equiv$13.98 €/MWh (right).

Outlook: The availability of a numerical approximation scheme allows us to extend our model to all market participants, so that regulatory constraints can be determined e.g. for reaching desired environmental goals. Ongoing work is concerned with model order reduction to make the scheme real-time efficient 24 hours a day with continuous incoming data (market and forecast).

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