

Aufgabe 4 (*Matrix norm and condition number*)

(12 Punkte)

We say that a matrix norm $\|\cdot\|_M$ is induced by a vector norm $\|\cdot\|_V$ if $\|A\|_M = \sup_{x \neq 0} \frac{\|Ax\|_V}{\|x\|_V}$.

(a) Denote by $\lambda_{\max}(A^T A)$ the largest eigenvalue of $A^T A$ and set $\|A\|_2 := \sqrt{\lambda_{\max}(A^T A)}$. Show that the Euclidian vector norm induces the matrix norm $\|A\|_2$.

(b) Let A be a regular matrix. Show $\|A^{-1}\|_2 = \frac{1}{\sqrt{\lambda_{\min}(A^T A)}}$.

(c) Calculate the condition number $\kappa_2(A) := \|A\|_2 \|A^{-1}\|_2$ for

$$A = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ 0 & -\sqrt{\frac{3}{2}} & \sqrt{\frac{3}{2}} \\ \sqrt{\frac{5}{2}} & -\frac{\sqrt{5}}{2} & \frac{\sqrt{5}}{2} \end{pmatrix}.$$