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Mathematics of Games

Sample Exam - Missing Solution

1. Consider the Copacabana restaurant game where a guest is an argentinian type with probability 0.5 or a german type with probability 0.5. The german would rather have a beer, while the argentinian would rather have a Chimarrão (sort of hot tea). The brazilian waiter loves to have a fight with argentinians, and in particular if the argentinian asks for a nice cold beer. On the other hand, the brazilian survivor and sympathy instincts tell the waiter it is better to treat very well the strong-built german, specially when germans prevented Argentinia from winning the World Cup in Maracanã. The payoffs are as in the table below. Give all pooling and separating Perfect Nash Equilibria. Which Equilibria are eliminated by the *Intuitive Criterion*, defined as seen in class, and which survive it?

(a_1, a_2)	Argentinian	German
$(Beer, not)$	2,0	3,2
$(Beer, duel)$	1,3	2,0
$(Chimarrao, not)$	3,3	2,0
$(Chimarrao, duel)$	0,1	1,1

Intuitive Criterion: Fix a vector of equilibrium payoffs u_1^* for player 1. For each strategy a_1, let $J(a_1)$ be the set of all θ such that $u_1^*(\theta) > \max u_1(a_1, a_2, \theta) \forall a_2 \in BR(T, a_1)$, where $BR(T, a_1)$ is the set of all pure-strategy best responses for player 2 to a_1 for beliefs $P(.|a_1)$ such that $P(T|a_1) = 1$. If for some a_1 there exists a $\theta' \in T$ such that $u_1^*(\theta) < \min u_1(a_1, a_2, \theta') \forall a_2 \in BR(T \setminus J(a_1), a_1)$, then the equilibrium fails this criterion.

(12 points)

Solution:

In this game, for instance, $((Chimarrao, Chimarrao)$,(duel,not),$p \leq \frac{3}{5}$,$q = \frac{1}{2})$ is a pooling P.B.E., where p and q are the probabilities that the guest is german given that Beer and Chimarrao are asked, respectively, $(Chimarrao, Chimarrao)$ are the strategies for the german and the argentinian types, and (duel,not) are the waiter decisions if Beer and Chimarrao are asked, respectively. Given the probabilities, neither player 1 nor player 2 will be tempted to deviate, as both would get at most the payoff from the equilibrium path.

Now, if the perfect bayesian equilibrium is pooling in Chimarrao, J(Beer) = Argentinian. If the probability for the Argentinian type choosing Beer is $1 - p = 0$ instead of $q \geq \frac{1}{2}$, then the German type would necessarily do better by choosing Beer ($3 > 2$), as the waiter would not duel in this case . Thus, according to definition, this equilibrium is not a very reasonable one as it fails the Intuitive Criterion, getting eliminated when one considers intuition on deviation patterns.

Besides, ((*Beer*,*Beer*),(duel,duel), $p = \frac{1}{2}$, $q \geq \frac{2}{3}$) is also a pooling P.B.E., where p and q are the probabilities that the guest is german given that Beer and Chimarrao are asked, respectively, (*Beer*,*Beer*) are the strategies for the german and the argentinian types, and (duel,duel) are the waiter decisions if Beer and Chimarrao are asked, respectively. Given the probabilities, neither player 1 nor player 2 will be tempted to deviate, as both would get at most the payoff from the equilibrium path.

Now, if the perfect bayesian equilibrium is pooling in Beer, J(Chimarrao) = $\emptyset$, as 2 is not greater than all Chimarrao payoffs for the german (if the waiter does not duel, Chimarrao gives 2 as well to the german) and 1 is not greater than all Chimarrao payoffs for the argentinian (who could get a 3). Thus, according to definition, this equilibrium is a very reasonable one.

Moreover, ((*Beer*,*Chimarrao*),(not,not), $p = 1$, $q = 0$) is a separating P.B.E., where p and q are the probabilities that the guest is german given that Beer and Chimarrao are asked, respectively, (*Beer*,*Chimarrao*) are the strategies for the german and the argentinian types, and (not,not) are the waiter decisions if Beer and Chimarrao are asked, respectively. Given the probabilities, neither player 1 nor player 2 will be tempted to deviate, as both would get less than the payoff from the equilibrium path.

Now, if the perfect bayesian equilibrium is separating in Beer for the german and Chimarrao for the argentinian, J(Chimarrao) = German and J(Beer) = Argentinian. Therefore, none would deviate. Thus, according to definition, this equilibrium is a very reasonable one as well.

Finally, ((*Chimarrao*,*Beer*),(duel,duel), $p = 0$, $q = 1$) is not a separating P.B.E., as the german would be tempted to deviate to Beer since $2 > 1$.