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Mathematics of Games

Exercise session 11

15.07.2013, 12pm-2pm, N24-H15

1. Create your own complete information simultaneous game with 2 players and 2 strategies for each player, with a non-pure mixed-strategy Nash Equilibrium as a possible solution, and then perturb its payoffs with incomplete information so that when the type interval space converges to a point, the Bayesian Equilibrium converges to the mixed-strategy Nash Equilibrium. The game must differ from Battle of the Sexes and Pick the Rose.

2. In the hotel guest in Bavaria game (Exerc. 2 of Exercise Sheet 10), we've seen that both ((Coffee,Coffee),(not,duel), $p = 0.1$, $q \geq \frac{1}{2}$) and ((Beer,Beer),(duel,not), $p \geq \frac{1}{2}$ $q = 0.1$) are pooling Perfect Bayesian Equilibria. Use an extra criterion, called the Intuitive Criterion, described as follows, to show that pooling in Coffee is not a very reasonable P.B.E (i.e., one eliminated by such a criterion), while pooling in Beer is a much more reasonable P.B.E. (i.e., one satisfying such a criterion).

   Intuitive Criterion: Fix a vector of equilibrium payoffs u_1^* for player 1. For each strategy a_1, let $J(a_1)$ be the set of all θ such that $u_1^*(\theta) > \max u_1(a_1, a_2, \theta) \forall a_2 \in BR(T, a_1)$, where $BR(T, a_1)$ is the set of all pure-strategy best responses for player 2 to action a_1 for beliefs $P(.|a_1)$ such that $P(T|a_1) = 1$. If for some a_1 there exists a $\theta' \in T$ such that $u_1^*(\theta) < \min u_1(a_1, a_2, \theta') \forall a_2 \in BR(T \setminus J(a_1), a_1)$, then the equilibrium fails the Intuitive Criterion.

   In words, $J(a_1)$ is the set of types who get less than their equilibrium payoff by choosing a_1, provided player 2 plays an undominated strategy. The equilibrium fails the Intuitive Criterion if there exists a type who would necessarily do better by choosing a_1 than in equilibrium as long as player 2's beliefs assign probability 0 to types in $J(a_1)$.

3. Consider in the hotel guest in Bavaria game (Exerc. 2 of Exercise Sheet 10) that the brazilian type holds probability 0.6 while the german type holds probability 0.4, as well as the new payoffs mapping new preferences as in the table below, where the bold and proud well-built waiter, up to any challenge but no massacre, would rather duel with the strong german as well as not duel with the skinny brazilian. Give all pooling and separating Perfect Nash Equilibria. Do the P.B.E. change if both probabilities equal 0.5?

(a_1, a_2)	Brazilian	German
$(Coffee, not)$	3,1	2,0
$(Coffee, duel)$	1,0	0,1
$(Beer, not)$	2,1	3,0
$(Beer, duel)$	0,0	1,1

4. Give the pure-strategy Perfect Bayesian Nash Equilibria for the tree of Exercise 3 from Exercise Sheet 5. What happens if the payoffs of player 2 for $(B, D, F) = 4$ and for $(B, E, F) = 1$ are exchanged, so that $(B, D, F) = 1$ and $(B, E, F) = 4$ for player 2? Give the subgame-perfect Nash Equilibria as well as the Perfect Bayesian Nash Equilibria.