Dr. Lucia Draque Penso
Dr. Jens Maßberg

Institut für Optimierung und Operations Research
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Mathematics of Games

Exercise session 12

16.07.2013, 10pm-12pm, N24-H13

1. Consider the bayesian public-good game with two players from the lecture, as in Exercise Sheet 9, but now as a repeated game with two periods ($t = 0, 1$). In the repeated version of the game, the strategy space for each player is $\{0, 1\}$ in each period. A strategy for player i is a pair consisting of $\sigma_i^0(1|c_i)$ (player i's probability of contributing in the first period when his cost is c_i) and $\sigma_i^1(1|h^1, c_i)$ (the probability that player i contributes in the second period when his cost is c_i and when the history is $h^1 \in \{00, 01, 10, 11\}$.

   Give a symmetric Perfect Bayesian Equilibrium, where the cost thresholds for a contribution, c^1 for the first period and c^2 for the second-period, are the same for both players. Analyse c^2 in terms of its own probability as well as of c^1 and $P(c^1)$, considering that during the first period either: neither player contributed, or both players contributed, or only one player contributed. Is there any necessary assumptions for such an equilibrium to exist? Give equations that relate both cost thresholds, making use of their probabilities. In such an equilibrium, is there less or at least the same contribution in the first period of the two-period game than in the one-period game?

2. *Monopoly Fight:* A single long-run incumbent drink firm faces potential entry by a series of short-run drink firms, each of which plays once but observes all previous play. Each period, a potential entrant decides whether to enter or stay out of a particular type of drink market. (Each entrant can enter only a single type of drink market, from an infinite number of possibilities, and the entrant's markets are distinct.) If the entrant stays out, the incumbent enjoys a monopoly in the market. If the entrant enters, the incumbent must choose whether to fight or to accomodate. The incumbent's payoffs are $a > 0$ if the entrant stays out, 0 if the entrant enters and the incumbent accomodates, and -1 if the entrant enters and the incumbent fights. The incumbent's objective is to maximize the discounted sum of its per-peiod payoff. Denote by δ the incumbent's discount factor. Each entrant has two possible types: tough and soft. Tough entrants always enter. A soft entrant has payoff 0 if it stays out, -1 if it enters and is fought, and $b > 0$ if it enters and the incumbent accomodates. Each entrant's type is private information, and each is tough with probability q, independent of the others. Give a value of δ and a condition so that in a infinite-period game, there is a Perfect Bayesian Equilibrium where the incumbent fights all entrants as long as it has never accomodated and accomodates entrants if it has ever accomodated at least one. Define the entrants' strategy profile for such an equilibrium: when is it better to stay out?

3. Show which Perfect Bayesian Nash Equilibria from Exercise 3 of Exercise Sheet 11 survive (and which succumb to) the Intuitive Criterion from Exercise 2 of Exercise Sheet 11.

4. Give a mixed-strategy Bayesian Nash Equilibrium for a first-price auction, defined both in lecture and in Exercise 2 from Exercise Sheet 8, but now with a discrete number of types instead of a continuum. Consider that the only possible two types are $\underline{\theta}$ and $\overline{\theta}$, with $\underline{\theta} < \overline{\theta}$. The valuations are independent. Denote by $\underline{p}$ and $\overline{p}$ the probability that the type equals $\underline{\theta}$ and $\overline{\theta}$, respectively. Assume that the seller's reservation price or minimum bid is lower than $\underline{\theta}$. Assume for your mixed-strategy Bayesian Nash Equilibrium that type $\underline{\theta}$ bids $\underline{\theta}$ and type $\overline{\theta}$ randomizes according to the continuous distribution $F(s)$ on $[\underline{s}, \overline{s}]$. Which should be the values of $\underline{s}$ and $\overline{s}$ in such a mixed-strategy Bayesian Nash Equilibrium? When does trade takes place?