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Mathematics of Games

Exercise session 1

29.04.2013, 12pm-2pm, N24-H15

Hand-in in PAIRS, before class starts!

1. Each of n players gets a unique marker color. Now each player i ($i \in \{1, \ldots, n\}$) simultaneously chooses his *position*: a real number $x_i \in [0, 1]$. All points in $[0, 1]$ are colored: Each point y gets player i's marker color for that i with x_i **closest to** y.

   If a point $y \in [0, 1]$ has the same distance to more than one player's position x_i, its color is determined at random (uniformly). If one position x_i is chosen by more than one player, the points with minimum distance to x_i are colored randomly (uniformly) with one of those players' marker colors. Each player wants to color a largest possible part of the interval with his marker color.

   (i) If there are two players, what are the pure-strategy Nash equilibria for?

   (ii) If there are three players, does a pure-strategy Nash equilibrium exist for? If so, give all pure-strategy NE and if not, explain why not.

   (iii) If there are n players, does a pure-strategy Nash equilibrium exist for? If so, give all pure-strategy NE and if not, explain why not. Is there any difference between an even and an odd number of players?

   (iv) If there are two players, is there any pure-strategy that strictly dominates another?

2. Suppose there are n firms in the Cournot oligopoly model. Let q_i denote the quantity produced by firm i, and let $Q = q_1 + \ldots + q_n$ denote the aggregate quantity on the market. Let P denote the market-clearing price and assume that inverse demand is given by $P(Q) = a - Q$ (assuming $Q < a$, else $P = 0$). Assume that the total cost of firm i from producing quantity q_i is $C_i(q_i) = cq_i$. That is, there are no fixed costs and the marginal cost is constant at c, where we assume $c < a$. Following Cournot, suppose that the firms choose their quantities simultaneously. What is the pure-strategy Nash equilibrium? What happens as n approaches infinity?

3. Consider the Cournot duopoly model where inverse demand is $P(Q) = a - Q$, with q_i denoting the quantity produced by firm i and $Q = q_1 + q_2$, but firms have asymmetric marginal costs: c_1 for firm 1 and c_2 for firm 2. Thus, the total cost for firm i is $C_i(q_i) = c_i q_i$. ($i \in \{1, 2\}$). What is the pure-strategy Nash equilibrium if $0 < c_i < a/2$ for each firm? What if $c_1 < c_2 < a$ but $2c_2 > a + c_1$?

4. In the following normal-form games, what strategies survive iterated elimination of strictly dominated strategies? What are the pure-strategy Nash equilibria?

(a)

	L	C	R
T	2,0	1,1	4,2
M	3,4	1,2	2,3
B	1,3	0,2	3,0

(b)

	A	B	C	D
E	6,3	3,7	2,5	1,5
F	1,1	4,3	3,2	2,2
G	8,1	3,3	2,6	6,2
H	10,6	2,4	1,3	5,9