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Mathematics of Games

Exercise Session 1

A class will be given on 28.04.2014, 12pm-2pm, in N24-H14.
Exercise Session 1 due on 05.05.2014, by 12:15pm, N24-H14.
Total : 20 Points
Hand-in individual!

1. Each of n players gets a unique marker color. Now each player i ($i \in \{1, \dots, n\}$) simultaneously chooses his *position*: a real number $x_i \in [0, 1]$. All points in $[0, 1]$ are colored: Each point y gets player i's marker color for that i with x_i **closest to** y.

   If a point $y \in [0, 1]$ has the same distance to more than one player's position x_i, its color is determined at random (uniformly). If one position x_i is chosen by more than one player, the points with minimum distance to x_i are colored randomly (uniformly) with one of those players' marker colors. Each player wants to color a largest possible part of the interval with his marker color.

   (i) If there are two players, does a pure-strategy Nash Equilibrium exist for? If so, is it unique? Either give a pure-strategy NE (with justification) or explain why one does not exist. [1 Points]

   (ii) If there are two players, which pure-strategy for player i strictly dominate $x_i = 0$? Which pure-strategy for player i strictly dominate $x_i = 1$? (The pure-strategy $s \in S_i$ strictly dominates $s' \in S_i$ if $u_i(s, s_{-i}) > u_i(s', s_{-i}) \forall s_{-i} \in \prod_{j \neq i} S_j$.) [1 Point]

   (iii) If there are three players, does a pure-strategy Nash Equilibrium exist for? If so, is it unique? Either give a pure-strategy NE (with justification) or explain why one does not exist. [2 Points]

   (iv) If there are $n \geq 4$ players, does a pure-strategy Nash Equilibrium exist for? If so, is it unique? Either give a pure-strategy NE (with justification) or explain why one does not exist. [4 Points]

   (v) If there are $n \geq 2$ players and the coloring rule is changed to 'Each point y gets player i's marker color for that i with x_i **most distant from** y.', does a pure-strategy Nash Equilibrium exist for? If so, is it unique? Either give a pure-strategy NE (with justification) or explain why one does not exist. [2 Points]

[10 Points]

2. Suppose there are $n \geq 2$ firms in the Cournot oligopoly model. Let q_i denote the quantity produced by firm i, and let $Q = q_1 + \ldots + q_n$ denote the aggregate quantity on the market. Let P denote the market-clearing price and assume that inverse demand is given by $P(Q) = a - Q$ (assuming $Q < a$, else $P = 0$). Assume that costs are assymetric: the total cost of firm i from producing quantity q_i is $C_i(q_i) = c_i q_i$. Following Cournot, suppose that the firms choose their quantities simultaneously. What is the pure-strategy Nash Equilibrium if $0 < c_i < a/n$ for each firm? Consider $n = 2$ at first and then $n \geq 3$.

[2 + 4 = 6 Points]

3. A seller has one indivisible unit of an object for sale. There are I potential buyers, or bids, with valuations $0 \leq v_1 \leq \ldots \leq v_l$ for the object, and these valuations are common knowledge. The bidders simultaneously submit bids $s_i \in [0, \infty)$. The highest bidder wins the object and pays the second bid (i.e., if he wins ($s_i > \max_{j \neq i} s_j$), bidder i has utility $u_i = v_i - \max_{j \neq i} s_j$, and the other bidders pay nothing (and therefore have utility 0). If several bidders bid the highest price, the good is allocated randomly among them. Give a pure-strategy Nash Equilibrium for this second-price auction. Would the Nash Equilibrium still be valid if the bidders would not know about one another's valuations?

[4 Points]