Dr. Lucia Draque Penso
Dr. Jens Maßberg

Institut für Optimierung und Operations Research
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Mathematics of Games

Exercise session 2

06.05.2013, 12pm-2pm, N24-H15

Hand-in in PAIRS, before class starts!

1. Each of n inheritors simultaneously chooses its own *inheritance share*: a real number $x_i \in [0,1]$, representing the share to be received by inheritor i out of a one million euros inheritance. If $\sum_{i=1}^{n} x_i \leq 1$, every inheritor i receives x_i, otherwise all receive 0.

   (i) Give all pure-strategy Nash equilibria.

   (ii) Is there any mixed-strategy Nash equilibria which is not pure?

   (iii) Is there any strategy that strictly dominates any other?

2. In the lecture, the Bertrand duopoly model with differentiated products was analyzed. The case of homogeneous products yields a stark conclusion. Suppose that the quantity that consumers demand from firm i is $a - p_i$ when $p_i < p_j$, 0 when $p_i > p_j$, and $(a - p_i)/2$ when $p_i = p_j$. Suppose also that there are no fixed costs and that marginal costs are constant at c, where $c < a$. Show that if the firms choose prices simultaneously, then the unique Nash equilibrium is that both firms charge the price c.

3. Consider a simultaneous-move auction in which two players simultaneously choose bids, which must be in nonnegative integer multiples of one dollar. The higher bidder wins a hundred dollar bill. If the bids are equal, neither player receives the bill. Each player must pay his own bid, whether or not he wins the bill. (The loser pays too.) Each player's utility is simply his net winnings. Construct a symmetric mixed-strategy equilibrium in which every bid less than one hundred has a positive probability.

4. Consider the game "rock-paper-scissors", defined by the following bi-matrix. Solve for the mixed-strategy Nash equilibria.

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0