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Sommersemester 2014

Mathematics of Games

Exercise Session 3

Exercise Session 3 due on 19.05.2014, by 12:15pm, N24-H14.
Total : 20 Points
Hand-in IN PAIRS!

1. Suppose that n oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + \ldots + q_n$ and q_i is the quantity produced by firm i. Each firm has a constant marginal cost of production, c, and no fixed cost. The firms choose their quantities in two periods as follows:

   (a) (1) firm 1 chooses $q_1 \geq 0$, (2) firms i, $2 \leq i \leq n$ observe the chosen value and then simultaneously choose q_i. [4 Points]

   (b) (1) firms j, $1 \leq j < z \leq n$, choose $q_j \geq 0$, (2) firms i, $z \leq i \leq n$, observe the chosen values and then simultaneously choose q_i. [4 Points]

   What is a subgame-perfect Nash Equilibrium for game (a)? What about for game (b)?

   [8 Points]

2. Two investors have each deposited D with a bank. The bank has invested these deposits in a long-term project. If the bank is forced to liquidate its investment before the project matures, a total of $2r$ can be recovered, where $D > r > D/2$. If the bank allows the investment to reach maturity, however, the project will pay out a total of $2R$, where $R > D$.
   There are two dates at which the investors can make withdrawals from the bank: date 1 is before the bank's investment matures; date 2 is after. For simplicity, assume that there is no discounting. If both investors make withdrawals at date 1 then each receives r and the game ends. If only one investor makes a withdrawal at date 1 then that investor receives D, the other receives $2r - D$, and the game ends.
   Finally, if neither investor makes a withdrawal at date 1 then the project matures and the investors make withdrawal decisions at date 2. If both investors make withdrawals at date 2 then each receives R and the game ends. If only one investor makes a withdrawal at date 2 then that investor receives $2R - D$, the other receives D, and the game ends. If neither investor makes a withdrawal at date 2 then the bank returns R to each investor and the game ends.
   Give all subgame-perfect Nash Equilibria for this 2-period bank-run dynamic game.

   [6 Points]

3. Consider the “battle of the sexes” game, as shown in the figure below.

	B	F
F	0,0	2,1
B	1,2	0,0

On a Friday evening, a couple prefers going out together (+1) than alone (0). Linda would prefer going to a Fight game (+1, but only if together with Peter), while Peter would prefer going to a Ball game (+1, but only if together with Linda). Let $\delta = \frac{9}{10}$, and consider the infinitely repeated game based on the stage game, with discount factor δ. Find a pure-strategy subgame-perfect Nash equilibrium s of the infinitely repeated game, for which the corresponding average discounted payoffs

$$(1-\delta) \cdot \sum_{t=0}^{\infty} \delta^t g_i(s^t(h^t))$$

for both players are both higher than $\frac{4}{3}$, double the highest symmetric payoff $(\frac{2}{3}, \frac{2}{3})$ of a NE for the stage game.

[6 Points]