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Mathematics of Games

Exercise session 4

Hand-in: 27.05.2013, 12pm-2pm, N24-H15

1. (a) Consider the original "battle of the sexes" game, as shown in the figure below.

	B	F
F	0,0	2,1
B	1,2	0,0

Let $\delta = \frac{9}{10}$, and consider the infinitely repeated game based on the stage game, with discount factor δ. Find a pure-strategy subgame-perfect Nash equilibrium s of the infinitely repeated game, for which the corresponding average discounted payoffs

$$(1-\delta) \cdot \sum_{t=0}^{\infty} \delta^t g_i(s^t(h^t))$$

for both players are both higher than $\frac{2}{3}$, the highest symmetric payoff $(\frac{2}{3}, \frac{2}{3})$ of a NE for the stage game. What about if the corresponding average discounted payoffs for both players are both higher than $\frac{4}{3}$, double the highest symmetric payoff of a NE for the stage game? Does the same s work? If not, give one.

2. Consider the infinitely repeated 2-player-game with discount factor δ, based on the stage game described by the figure below.

	A	B
A	1,1	6,0
B	0,6	3,3

Assume that each player plays the following strategy:

"Play B in the first stage. In the t^{th} stage, if the outcome of all $t-1$ preceeding stages has been (B, B), then play B, otherwise, play A."

Compute the values of δ for which this stragegy for both firms is a subgame-perfect NE.

3. The accompanying simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. The variable x is greater than 4, so that $(4, 4)$ is not an equilibrium payoff in the one-shot game. For which values of x is the following strategy (played by both players) a subgame-perfect NE?

Play Q_i in the first stage. If the first-stage outcome is (Q_1, Q_2), play P_i in the second stage. If the first-stage outcome is (y, Q_2) where $y \neq Q_1$, play R_i in the second stage. If the first-stage outcome is (Q_1, z) where $z \neq Q_2$, play S_i in the second stage. If the first-stage outcome is (y, z) where $y \neq Q_1$ and $z \neq Q_2$, play P_i in the second stage.

	P_2	Q_2	R_2	S_2
P_1	2,2	x,0	-1,0	0,0
Q_1	0,x	4,4	-1,0	0,0
R_1	0,0	0,0	0,2	0,0
S_1	0,-1	0,-1	-1,-1	2,0

4. Draw game trees for the following games and solve for the subgame-perfect Nash Equilibria.

   (a) 1. Player 1 chooses an action a_1 from the feasible set $A_1 = \{A, D\}$ where A ends the game with payoffs of 2 to player 1 and 0 to players 2 and 3.
   2. Player 2 observes a_1 and if $a_1 = D$, player 2 chooses an action a_2 from the feasible set $A_2 = \{L, R\}$.
   3. Player 3 observes a_1 (but not a_2) and if $a_1 = D$, player 3 chooses an action a_3 from the feasible set $A_3 = \{L', R'\}$, which ends the game with payoffs given in the table below.

(a_1, a_2, a_3)	Player 1	Player 2	Player 3
(D, L, L')	1	2	1
(D, L, R')	3	3	3
(D, R, L')	0	1	1
(D, R, R')	0	1	2

   (b) 1. Player 1 chooses an action a_1 from the feasible set $A_1 = \{A, L, R\}$ where A ends the game with payoffs of 20 to player 1 and 1 to players 2 and 3.
   2. Player 2 observes a_1. If $a_1 = L$, player 2 chooses an action a_2 from the feasible set $A_{2_L} = \{L', R'\}$ and if $a_1 = R$, player 2 chooses an action a_2 from the feasible set $A_{2_R} = \{A', L', R'\}$, where A' ends the game with payoffs 18 for player 1, 8 for player 2 and 6 for player 3.
   3. Player 3 observes if $a_1 \neq A$ and if so, player 3 observes if $a_2 \neq A'$. If $a_1 \neq A$ and $a_2 \neq A'$, player 3 observes whether or not $(a_1, a_2) = (R, R')$ and then chooses an action a_3 from the feasible set $A_3 = \{L'', R''\}$, which ends the game with payoffs given in the table below.

(a_1, a_2, a_3)	Player 1	Player 2	Player 3
(L, L', L'')	20	8	4
(L, L', R'')	8	0	1
(L, R', L'')	4	4	5
(L, R', R'')	2	6	1
(R, L', L'')	12	8	2
(R, L', R'')	16	4	1
(R, R', L'')	10	2	5
(R, R', R'')	20	10	6