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Mathematics of Games

Exercise Session 4

Exercise Session 4 due on 26.05.2014, by 12:15pm, N24-H14.
Total : 20 Points
Hand-in IN PAIRS!

1. Solve Rubinstein-Ståhl's finite-horizon bargaining problem for T even and then for T odd, and show that the outcomes of the two cases converge to a common limit as $T \to \infty$.

   [6 Points]

2. Consider the following infinitely repeated game with discount factor δ, based on the Cournot 2-firm-game with symmetric cost as stage game. Assume that both firms play:

   "Produce half the monopoly quantity, $q_m/2 = (a-c)/4$, in the first period. In the t^{th} period, produce $q_m/2$ if both firms have produced $q_m/2$ in each of the $t-1$ previous periods; otherwise, produce the Cournot quantity, $q_C = (a-c)/3$."

   For which values of δ is the above stragegy a subgame-perfect Nash Equilibrium?

   [5 Points]

3. Consider the infinitely repeated 2-player-game with discount factor δ, based on the stage game described by the figure below.

	A	B
A	1,1	6,0
B	0,6	3,3

   Assume the following strategy, where each player:

   "Play B in the first stage. In the t^{th} stage, if the outcome of all $t-1$ preceeding stages has been (B, B), then play B, otherwise, play A."

   For which values of δ is this stragegy a subgame-perfect Nash Equilibrium?

   [5 Points]

4. The accompanying simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. The variable x is greater than 4, so that $(4, 4)$ is not an equilibrium payoff in the one-shot game. For which values of x is the following strategy (played by both players) a subgame-perfect NE?

   Play Q_i in the first stage. If the first-stage outcome is (Q_1, Q_2), play P_i in the second stage. If the first-stage outcome is (y, Q_2) where $y \neq Q_1$, play R_i in the second stage. If the first-stage outcome is (Q_1, z) where $z \neq Q_2$, play S_i in the second stage. If the first-stage outcome is (y, z) where $y \neq Q_1$ and $z \neq Q_2$, play P_i in the second stage.

	P_2	Q_2	R_2	S_2
P_1	2,2	x,0	-1,0	0,0
Q_1	0,x	4,4	-1,0	0,0
R_1	0,0	0,0	0,2	0,0
S_1	0,-1	0,-1	-1,-1	2,0

[4 Points]