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# Mathematics of Games

## Exercise session 5

03.06.2013, 12pm-2pm, N24-H15

Hand-in in PAIRS, before class starts!

1. Solve Rubinstein-Ståhl's finite-horizon bargaining problem for $T$ even and then for $T$ odd, and show that the outcomes of the two cases converge to a common limit as $T \to \infty$.

2. Consider the Cournot game for two firms known from the lecture. Consider the infinitely repeated game based on this stage game. Assume that both firms play the following strategy:

   "Produce half the monopoly quantity, $q_m/2 = (a-c)/4$, in the first period. In the $t^{th}$ period, produce $q_m/2$ if both firms have produced $q_m/2$ in each of the $t-1$ previous periods; otherwise, produce the Cournot quantity, $q_C = (a-c)/3$."

   Compute the values of $\delta$ for which the above stragegy for both firms is a subgame-perfect Nash equilibrium.

3. Draw a game tree for the following game and solve for the subgame-perfect Nash equilibria. Are there Nash equilibria which are not subgame-perfect? If yes, give one.

   1. Player 1 chooses an action $a_1$ from the feasible set $A_1 = \{B, C\}$.
   2. Player 2 observes $a_1$ and chooses an action $a_2$ from the feasible set $A_2 = \{D, E\}$. If $a_1 = C$, the game ends after player 2's action, with payoffs $(2, 0, 3)$ if $a_2 = D$ and $(0, 2, 1)$ if $a_2 = E$.
   3. Player 3 observes $a_1$ (but not $a_2$) and if $a_1 = B$, player 3 chooses an action $a_3$ from the feasible set $A_3 = \{F, G\}$, which ends the game with payoffs given in the table below.

| $(a_1, a_2, a_3)$ | Player 1 | Player 2 | Player 3 |
|---|---|---|---|
| $(B, D, F)$ | 2 | 4 | 3 |
| $(B, D, G)$ | 1 | 5 | 0 |
| $(B, E, F)$ | 0 | 1 | 2 |
| $(B, E, G)$ | 3 | 0 | 1 |