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# Mathematics of Games

## Exercise session 6

10.06.2013, 12pm-2pm, N24-H15

Hand-in before class starts.

1. Consider a variant of Rubinstein's infinite-horizon bargaining game where partitions $(x_1, x_2)$, with $x_1 + x_2 = 1$, are restricted to be integer multiples of 0.01, that is, $x_i$ can be $0, 0.01, 0.02, \ldots, 0.99$, or 1 for $i = 1, 2$. There is a common discount factor $\delta$. Prove that, if $\delta > 0.99$, any partition can be supported as a subgame-perfect NE.

   *Hint:* If player $i$ always offers $(x_i, x_j)$, which would be the threshold value $t$ for $i$ to accept an offer $(y_i, y_j)$ from $j$? Can player $i$ gain by making an offer with a value lower or higher than $x_i$ for its partition? Can player $i$ lose by accepting any offer from $j$ of at least $t$ or by rejecting any offer from $j$ less than $t$?

2. Consider the static Bertrand duopoly model (with homogeneous products): The two firms name prices sinultaneously; demand for firm $i$'s product is $a - p_i$ if $p_i < p_j$, is 0 if $p_i > p_j$, and is $(a - p_i)/2$ if $p_i = p_j$; marginal costs are $c < a$. Consider the infinitely repeated game based on this stage game. Assume that both players play the following strategy:

   "Play price $\frac{a+c}{2}$ in the first period. In the $t^{th}$ period, play price $\frac{a+c}{2}$ if both played $\frac{a+c}{2}$ in each of the $t-1$ previous periods, otherwise, play price $c$."

   Show that the above strategy for both firms is a subgame-perfect Nash equilibrium if and only if $\delta \geq 1/2$.

3. Three players bargain over the division of a pie of size 1. A division is a triple $(x_1, x_2, x_3)$ of shares for each player, where $x_i \geq 0$, $\sum_{i=1}^{3} x_i = 1$. At dates $3k+1, k = 0, 1, \ldots$, player 1 offers a division, then players 2 and 3 simultaneously decide whether they accept or veto. If players 2 and 3 both accept, the game is over. If one or both of them veto, bargaining goes on. Similarly, at dates $3k + 2$ (respectively, $3k$), player 2 (respectively, player 3) makes the offer. The game stops once an offer by one player has been accepted by the other two players. The players have common discount factor $\delta$.

   Show that for every $\delta \in (0, 1)$, any partition can be supported as a subgame-perfect Nash equilibrium.