Dr. Lucia Draque Penso
Dr. Jens Maßberg

ulm university universität ulm
Institut für Optimierung und Operations Research
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Mathematics of Games

Exercise Session 6

Exercise Session due on Thursday, 12.06.2014, by 14:15pm, N24-H15.
Total : 20 Points
Hand-in IN PAIRS!
Let the games begin! :-) Enjoy World Cup 2014! :-)

1. *Cat-Dog Fight Game*: A cat and a dog are fighting for a prize whose current value at any time $t = 0, 1, \ldots$ is $v > 1$. Fighting costs 1 unit per period. If only one animal stops fighting at period t, his opponent wins alone the prize without incurring a fighting cost that period, and the choice of the second stopping time is irrelevant. If both animals stop fighting simultaneously, neither one wins the prize. That is, if we consider a per-period discount factor δ, the (symmetric) payoff functions are:

   - $L(t) = -(1 + \delta + \ldots + \delta^{t-1}) = -\frac{1-\delta^t}{1-\delta}$, for the loser(s), and
   - $W(t) = L(t) + \delta^t v$, for the winner (in case there is one).

   Give a symmetric subgame-perfect Nash Equilibrium for the *Cat-Dog Fight Game*.

   Hint: You may consider a mixed strategy profile.

   [5 Points]

2. Three players bargain over the partition of a pie of size 1. A partition is a triple (x_1, x_2, x_3) of shares for each player, where $x_i \geq 0$, $\sum_{i=1}^{3} x_i = 1$. At dates $3k+1$, $k = 0, 1, \ldots$, player 1 offers a division, then players 2 and 3 simultaneously decide whether they accept or veto. If players 2 and 3 both accept, the game is over. If one or both of them veto, bargaining goes on. Similarly, at dates $3k+2$ (respectively, $3k$), player 2 (respectively, player 3) makes the offer. The game stops once an offer by one player has been accepted by the other two players. The players have common discount factor δ. Show that, for every $\delta \in (0, 1)$, all partitions can be supported as a subgame-perfect Nash Equilibrium.

   [5 Points]

3. Consider a Cournot duopoly operating in a market with inverse demand $P(Q) = a - Q$, where $Q = q_1 + q_2$ is the aggregate quantity on the market. Both firms have total costs $c_i(q_i) = cq_i$ with a constant c, but demand is uncertain: it is high ($a = a_H$) with probability γ and low ($a = a_L$) with probability $1-\gamma$. So the payoff depends on a and is $u_i(q_i, q_j, a) = (P(Q)-c)q_i$ for both firms. Furthermore, information is asymmetric: firm 1 knows whether demand is high or low, but firm 2 does not. All of this is common knowledge. The two firms simultaneously choose quantities. What is the pure-strategy Bayesian Nash equilibrium of this static Bayesian game?

[5 Points]

4. Consider the following model of Bertrand duopoly with differentiated products. Demand for firm i is $q_i(p_i, p_j) = a - p_i - b_i \cdot p_j$. Costs are zero for both firms. The sensitivity of firm i's demand to firm j's price is either high or low. That is, b_i is either b_H or b_L, where $b_H > b_L > 0$. For each firm, $b_i = b_H$ with probability γ and $b_i = b_L$ with probability $1-\gamma$, independent of the realization of b_j. Each firm knows its own b_i but not its competitor's. All of this is common knowledge. Which (four) conditions define a pure-strategy Bayesian Nash Equilibrium of this game? What is the pure-strategy Bayesian Nash Equilibrium in the specific case $\gamma = \frac{1}{2}$? What about if $\gamma = 1$ or if $\gamma = 0$?

[5 Points]