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Mathematics of Games

Exercise Session 7

Exercise Session due on Monday, 16.06.2014, by 12:15pm, N24-H14.
Total : 20 Points
Hand-in IN PAIRS!

1. Consider a first-price, sealed-bid auction in which the bidders' valuations are independently and uniformly distributed on $[0, 1]$. Bidder i has valuation v_i for the good - that is, if bidder i gets the good and pays the price p, then i's payoff is $v_i - p$. The bids $b_i \geq 0$ are submitted simultaneously, the higher bidder wins the good and pays the bidden price, the other bidders get and pay 0. In case of a tie, the winner is determined uniformly at random. Show that if there are n bidders, then the strategy of bidding $\frac{n-1}{n}$ times one's valuation is a Bayesian Nash equilibrium of this auction.

   [6 Points]

2. Give a mixed-strategy Bayesian Nash Equilibrium for a first-price auction, as defined before, but now with a discrete number of types instead of a continuum. Consider that the only possible two types are $\underline{\theta}$ and $\overline{\theta}$, with $\underline{\theta} < \overline{\theta}$. The valuations are independent. Denote by $\underline{p}$ and $\overline{p}$ the probability that the type equals $\underline{\theta}$ and $\overline{\theta}$, respectively. Assume that the seller's reservation price or minimum bid is lower than $\underline{\theta}$. Assume for your mixed-strategy Bayesian Nash Equilibrium that type $\underline{\theta}$ bids $\underline{\theta}$ and type $\overline{\theta}$ randomizes according to the continuous distribution $F(s)$ on $[\underline{s}, \overline{s}]$. Which should be the values of $\underline{s}$ and $\overline{s}$ in such a mixed-strategy Bayesian Nash Equilibrium? When does trade takes place?

   [6 Points]

3. Consider the public-good game from the lecture. Suppose that there are $n > 1$ players and that the public good is supplied (with benefit 1 for all players) only if at least $K \in \{2, \ldots, n\}$ players contribute. The players' costs of contributing, $\theta_1, \ldots, \theta_n$, are independently drawn from the distribution $P(\cdot)$ on $[\underline{\theta}, \overline{\theta}]$ where $\underline{\theta} < 1 < \overline{\theta}$. Show that there always exists a trivial equilibrium in which nobody contributes. (Assume $\underline{\theta} > 0$.) Derive a second, more interesting Bayesian equilibrium, where player i contributes. When it is optimal for player i to contribute? Give a range of the types of player i which contribute.

   [4 Points]

4. Consider the following "matching coins variation" game, where both players win when having the same face whereas both players lose when having distinct faces, as shown in the figure below. Draw a best-response graphic with probabilities from strategy profile.

	H	T
H	1,1	-1,-1
T	-1,-1	1,1

Modify this game of complete information into a game of incomplete information with a continuum of types, so that whenever the type interval tends to a point, the Bayesian Nash Equilibrium tends to a Nash Equilibrium of the complete information game. Give the transformations, as well as the respective (Bayesian) Nash Equilibria.

[4 Points]