Dr. Lucia Draque Penso
Dr. Jens Maßberg

Institut für Optimierung und Operations Research
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Mathematics of Games

Exercise session 8

01.07.2014, 10am-12pm, N24-H13

Hand-in IN PAIRS!

1. Two players are fighting for a gorgeous pink rose. If a player is able to ignore the charming rose and does not give it a try, he ends with nothing, without losing or winning. If both players pick the pink rose at the same time, the rose is destroyed and both lose with lots of thorns in the hands. However, if just a single player is smart enough to get the lovely rose alone, he wins her. Consider the following payoff table representation for this simultaneous one-shot complete information game with R = picking rose and I = ignoring rose.

(a_1, a_2)	Player 1	Player 2
(R, R)	-1	-1
(R, I)	1	0
(I, R)	0	1
(I, I)	0	0

Modify the winner payoff of 1 for player i with its type θ_i independently and uniformly distributed in $[-\theta, \theta]$, so that the one-shot game is one of incomplete information. Give a pure-strategy Bayesian Nash Equilibrium and show that when θ converges to 0 the pure-strategy Bayesian Nash Equilibrium converges to the mixed-strategy Nash Equilibrium of the complete information game.

[4 Points]

2. Consider the following signaling game: an entrepreneur needs outside financing of I to expanding. The entrepreneur has private information about the profitability of the existing company, either high ($\theta = H$) with probability $1 - p$ or low ($\theta = L$, where $H > L > 0$) with probability p. The financing of I would add an extra profit of R to it. The investor's alternative of return is r, and $R > I(1 + r)$. Suppose that the entrepreneur offers a potential investor an equity stake (i.e., a share) of s in the firm in exchange of the necessary financing, which the investor may reject or accept. What should be the equity stake of s? Give both a pooling as well as a separating Perfect Bayesian Nash Equilibrium.

[6 Points]

3. In the following signaling game, the first player is a hotel guest in Bavaria, of private type t either *german* with probability 0.9 or *brazilian* with probability 0.1, known by the first player but not the second one. The german type prefers to have its national drink (*beer*) at breakfast, while the brazilian type prefers to have its national drink (*coffee*) at breakfast, both types preferring not to duel with the hotel breakfast waiter, the second player, rather than receiving its preferred breakfast drink. The hotel breakfast waiter, on the other hand, may *duel* or *not* with the hotel guest's choice, and it would prefer to duel with the skinny brazilian type but not to duel with the strong-built german type. These preferences may be described in the following table, the first integer representing the payoff for the first player, and the second integer representing the payoff for the second player. For instance, if the waiter decides not to duel, if the brazilian guest type had asked for coffee or if the german guest type had asked for beer, the guest will receive a payoff of 3 while the waiter will receive a payoff of 0. Which are the (pooling and separating) Perfect Bayesian Nash Equilibria?

(a_1, a_2)	$t =$ Brazilian	$t =$ German
$(Coffee, Duel)$	1,1	0,-1
$(Coffee, Not)$	3,0	2,0
$(Beer, Duel)$	0,1	1,-1
$(Beer, Not)$	2,0	3,0

[6 Points]

4. Use an extra criterion in the above hotel guest in Bavaria game, called the Intuitive Criterion, described as follows, to show which pooling P.B.E. is not very reasonable (i.e., one eliminated by such a criterion), and which pooling P.B.E. is much more reasonable P.B.E. (i.e., one satisfying such a criterion).

   Intuitive Criterion: Fix a vector of equilibrium payoffs u_1^* for player 1. For each strategy a_1, let $J(a_1)$ be the set of all θ such that $u_1^*(\theta) > \max u_1(a_1, a_2, \theta) \forall a_2 \in BR(T, a_1)$, where $BR(T, a_1)$ is the set of all pure-strategy best responses for player 2 to action a_1 for beliefs $P(.|a_1)$ such that $P(T|a_1) = 1$. If for some a_1 there exists a $\theta' \in T$ such that $u_1^*(\theta) < \min u_1(a_1, a_2, \theta') \forall a_2 \in BR(T \setminus J(a_1), a_1)$, then the equilibrium fails the Intuitive Criterion.

   In words, $J(a_1)$ is the set of types who get less than their equilibrium payoff by choosing a_1, provided player 2 plays an undominated strategy. The equilibrium fails the Intuitive Criterion if there exists a type who would necessarily do better by choosing a_1 than in equilibrium as long as player 2's beliefs assign probability 0 to types in $J(a_1)$.

[4 Points]