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Mathematics of Games

Exercise session 9

01.07.2013, 12pm-2pm, N24-H15

Hand-in before class starts!

1. Consider the public-good game from the lecture. Suppose now that there are $n > 2$ players and that the public good is supplied (with benefit 1 for all players) only if at least $K \in \{1, \ldots, n\}$ players contribute. The players' costs of contributing, $\theta_1, \ldots, \theta_n$, are independently drawn from the distribution $P(\cdot)$ on $[\underline{\theta}, \overline{\theta}]$, where $\underline{\theta} < 1 < \overline{\theta}$.
   (a) Give a Bayesian Nash Equilibrium when $K = 1$, a generalization with $n > 2$ players of the public-good game of the lecture.
   (b) Suppose $K \geq 2$. Show that there always exists a trivial equilibrium in which nobody contributes. (Assume $\underline{\theta} > 0$.) Derive a second, more interesting Bayesian Nash Equilibrium, in which the public good is supplied. When is it optimal for player i to contribute there? Give a range for the types of player i which contribute in such a Bayesian Nash Equilibrium.

2. Two players are fighting for a gorgeous pink rose. If a player is able to ignore the charming rose and does not give it a try, he ends with nothing, without losing or winning. If both players pick the pink rose at the same time, the rose is destroyed and both lose with lots of thorns in the hands. However, if just a single player is smart enough to get the lovely rose alone, he wins her. Consider the following payoff table representation for this simultaneous one-shot complete information game with R = picking rose and I = ignoring rose.

(a_1, a_2)	Player 1	Player 2
(R, R)	-1	-1
(R, I)	1	0
(I, R)	0	1
(I, I)	0	0

Modify the winner payoff of 1 for player i with its type θ_i independently and uniformly distributed in $[-\theta, \theta]$, so that the one-shot game is one of incomplete information. Give a pure-strategy Bayesian Nash Equilibrium and show that when θ converges to 0 the pure-strategy Bayesian Nash Equilibrium converges to the mixed-strategy Nash Equilibrium of the complete information game.

3. Consider a double auction of two players with incomplete information, as described. A seller asks an amount of s for an item simultaneously with an offer b made by a buyer. If $b \geq s$, then the trade occurs at price $p = \frac{s+b}{2}$, otherwise it does not. Consider v_b and v_s to be the private valuations for the buyer and the seller respectively, drawn from independent uniform distributions on [0,1]. If there is trade, $v_b - p$ is the payoff for the buyer, while $p - v_s$ is the one for the seller. Otherwise, both get 0 as payoff. In this static bayesian game, a strategy $b(v_b)$ for the buyer is a function specifying the offer b for each possible valuation. Similarly, a strategy for the seller $s(v_s)$ is a function specifying the asked amount of s for each possible valuation. Give a linear Bayesian Nash Equilibrium. When does trade occur?

4. Draw a game tree for the following extensive-form game and give all pure-strategy subgame-perfect Nash Equilibria and all pure-strategy Perfect Bayesian Nash Equilibria.

1 Player 1 chooses a strategy a_1 from the set $A_1 = \{L, M, R\}$ of feasible strategies, where R ends the game with payoffs 2 for both players.

2 Player 2 observes if $a_1 = R$ and if not, he believes that player 1 played L with probability p and M with probability $1 - p$ and chooses a strategy a_2 from the feasible set $A_2 = \{L', R'\}$ which ends the game with payoffs given in the table below.

(a_1, a_2)	Player 1	Player 2
(L, L')	4	1
(L, R')	0	0
(M, L')	3	0
(M, R')	0	1

Hint: Calculate the expected payoff of player 2 from playing L' and R', given the probability of p that player 1's type is L and of $1 - p$ that it is M. The Perfect Bayesian Nash Equilibria happen within which range of p?