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Mathematics of Games

Exercise session 9

08.07.2014, 10am-12pm, N24-H13

Hand-in IN PAIRS!

1. Consider the following two-period game involving monetary policy under distinct inflation fighting types. In the monetary policy one-period stage game, first employers form an expectation of inflation, π_e, and then the monetary authority observes this expectation and chooses actual inflation, π_m. The monetary authority's one-period payoff is given by $u_m(\pi_m, \pi_e) = -c\pi_m^2 - ((b-1)o^* + d(\pi_m - \pi_e))^2$, while for employers, the one-period payoff is given by $u_e(\pi_m, \pi_e) = -(\pi_m - \pi_e)^2$. Payoffs sum over periods. The parameter $d > 0$ measures effect of surprise inflation on output o, while $b < 1$ reflects the presence of monopoly power in product markets. The monetary authority would like inflation to be zero but output (o) to be at its efficient level (o^*). The parameter c in the payoff function reflects the monetary authority's trade-off between the goals of zero inflation and efficient output. Suppose that this parameter is privately known by the monetary authority: either "$c = S$" or "$c = W$" (for strong or weak at fighting inflation, respectively) where $S > W > 0$ and the probability that "$c = S$" is p. Timing is as follows:

   - Employers expect the initial first-period inflation π_e^1 to be π if both types are initially expected to act the same (i.e., in a pooling fashion) and $(1-p)\pi_W + p\pi_S$ if both types are initially expected to act differently (i.e., in a separating fashion).
   - The monetary authority observes π_e^1 and chooses the first-period inflation π_m^1.
   - Employers observe π_m^1 (but not c) and form a new expectation of inflation, π_e^2.
   - The monetary authority observes π_e^2 and chooses a new second-period inflation π_m^2.

   If the monetary authority's type is c, which is it optimal choice of π_m given that the employers' expectation is π_e? Which are then the conditions needed on the equilibrium path for an initially (in the first period) pooling Perfect Bayesian Nash Equilibrium to exist? What about the conditions on the equilibrium path for an initially (in the first period) separating one?

   Note that there is a one-period signaling game embedded in this two-period monetary-policy game. The sender's message is π_m^1, and the receiver's action is π_e^2.

This game captures a scenario where employers and workers negotiate nominal wages, after which the monetary authority chooses the money suply, which in turn determines the rate of inflation. If wage contracts cannot be perfectly indexed, employers and workers will try to anticipate inflation in setting the wage. Once an imperfectly indexed nominal wage has been set, however, actual inflation will erode the real wage, causing employment and output to be expanded. The monetary authority thus faces a trade-off between the costs of inflation and the benefits of some surprise inflation. Nothing in excess is good!

[8 Points]

2. Use the Intuitive Criterion from Exercise Session 8, defined as follows, in the Spencer's educational game seen in class. Are the pooling and separating Perfect Bayesian Nash Equilibria there derived rather reasonable or unreasonable?

   Intuitive Criterion: Fix a vector of equilibrium payoffs u_1^* for player 1. For each strategy a_1, let $J(a_1)$ be the set of all θ such that $u_1^*(\theta) > \max u_1(a_1, a_2, \theta) \forall a_2 \in BR(T, a_1)$, where $BR(T, a_1)$ is the set of all pure-strategy best responses for player 2 to action a_1 for beliefs $P(.|a_1)$ such that $P(T|a_1) = 1$. If for some a_1 there exists a $\theta' \in T$ such that $u_1^*(\theta) < \min u_1(a_1, a_2, \theta') \forall a_2 \in BR(T \setminus J(a_1), a_1)$, then the equilibrium fails the Intuitive Criterion.

   In words, $J(a_1)$ is the set of types who get less than their equilibrium payoff by choosing a_1, provided player 2 plays an undominated strategy. The equilibrium fails the Intuitive Criterion if there exists a type who would necessarily do better by choosing a_1 than in equilibrium as long as player 2's beliefs assign probability 0 to types in $J(a_1)$.

   [6 Points]

3. In the alsacian signaling game seen in class, we've seen a pooling P.B.E. on "Foie Gras", as well as verified that there is no pooling P.B.E. on "Sausage". Are there any separating Perfect Bayesian Nash Equilibria? Are the P.B.E. for this game rather more reasonable or more unreasonable under the Intuitive Criterion?

   [6 Points]