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Wintersemester 2014/2015

Online and Distributed Algorithms

Exercise Session 10

1. *MIS and Dominating Set:* Every MIS is a dominating set.

   (a) How much larger however can it be compared to the size of an optimal minimum dominating set? Give an example.

   (b) A graph is said to be of bounded independence if each neighbourhood contains at most a constant number of independent (i.e., mutually non-adjacent) nodes. Show that in graphs of bounded independence, a constant-factor approximation to a minimum dominating set can be found in $O(\log n)$ time with high probability. (An approximation algorithm $\mathcal{A}$ for an optimization problem II has a factor of r if for all its instances $I \in II$, $\frac{OPT(I)}{\mathcal{A}(I)} \leq r$.)

   (c) Are there other problems that can be solved with the help of MIS? Show that one can $\Delta + 1$-color an arbitrary graph in $O(\log n)$ time with high probability, Δ being the largest degree in the graph.

2. *k-Domination and k-Distance:* Consider the following game in a graph G between two players. Player 1 chooses first a node p_1 in G and then player 2 chooses a node $p_2 \neq p_1$ in G. If a node v has distance k_1 to p_1 and the same node v has distance k_2 to p_2, v belongs only to player 1 if $k_1 < k_2$, only to player 2 if $k_2 < k_1$, and to both if $k_1 = k_2$. The objective of each player is to maximize the number of owned nodes in G. Note that each player i owns at least p_i. In many graphs (but not all) it is an advantage for the first player to decide first. To see that, consider for instance, a star and its center. Show a graph where the second player always wins, independent of the decision of the first player.