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Online and Distributed Algorithms

Exercise Session 1

1. The relationship between knowledge, action and (synchronous or asynchonous) communication is an essencial one in distributed computing. In the following monk puzzle, the following message is sent simultaneously by a holy person to all monks of a monestary: "There are (one or more) guilty monks who committed the sin of breaking the sacred rules of confession confidentiality in our community. Although none of you knows if you are guilty, each one of you knows if the others are guilty or not. I forbid you to discuss the matter of your own guiltiness with anyone. However, if you discover you are guilty, you must shoot yourself on the midnight of the day you find out about it." Suppose every monk can perfectly hear another monk shooting himself.

   (i) If messages are synchronous (always arriving before midnight of the same day), and there had been n guilty monks at the time the holy person simultaneously sent the message to all monks, how many days it would take until all guilty monks would have shot themselves? In which days shootings would have ocurred?

   (ii) What about if messages are asynchronous (arriving eventually, in a finite but unbounded amount of time)? Would any guilty monk still die?

   (iii) What about if messages are asynchonous but instead of sending simultaneously messages to all monks, a single message is sent over a path formed with the monks: the holy person sends a single message to monk 1, and when monk i receives it, it gets forwarded only to monk $i+1$, and so on, until it reaches the last monk m, supposing monks are numbered from 1 to m. Is this scenario distinct to the previous ones? Would any guilty monk die? If so, which exactly?

2. Consider Algorithm 3 seen in class.

   (i) If "undecided" messages are neglected, what is the worst message complexity of Algorithm 3? Note that such messages are not necessary, as an absence of a message can be interpreted as "undecided" in the synchronous scenario.

   (ii) Is there a way to modify Algorithm 3 so that it would work in an asynchronous environment? If so, would "undecided" messages be necessary?