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Online and Distributed Algorithms

Exercise Session 3

1. *The Generals' Valley War:* Two generals are fighting together for a territory in a valley, which has been taken by a strong enemy and its large army. Each general stands on an opposite hilly side of the enemy-taken valley territory. Both generals need to attack simultaneously at the very same exact time in order to win, as the enemy is considerably larger and more powerful. To communicate, each general needs to send a scout to the other side, who needs to cross the valley. Scouts are very smart and never die or get caught, however, due to the enemy's presence, there is no way to predict how long it might take for them to reach the hill on the other valley's side. Prove that, no matter how many (a finite number of) scouts are sent back and forth, there is no way of both generals agreeing on a exact time to beat the enemy. (Tip: Use an inductive argument).

2. *Spy $\times$ Spy:* Lucia decided to build up her own Secret Service agency (LSS) and wants to create a tree hierarchy with her as the root, given a connected graph of trust with n selected names. Give an asynchronous algorithm so that individuals get as close to her in the Secret Service agency tree hierarchy as in the graph. Which complexities do you get?

3. *License to Spy Pairing:* In preparation of a highly dangerous mission, the participating agents of Lucia's Secret Service (LSS) need to work in pairs of two for safety reasons. All members in the LSS are organized in a tree hierarchy. Communication is only possible via the official channel: an agent has a secure phone line to his direct superior and a secure phone line to each of his direct subordinates. Initially, each agent knows whether or not he is taking part in this mission. The goal is for each agent to find a partner.

   (i) Devise an algorithm that will match up a participating agent with another participating agent given the constrained communication scenario. A "match" consists of an agent knowing the identity of his partner and the path in the hierarchy connecting them. Assume that there is an even number of participating agents so that each one is guaranteed a partner.

   (ii) What are the time and message (i.e., "phone call") complexities of your algorithm?

4. *License to Spy Sharing:* We consider another day at LSS. After the above mission was successful, the involved agents collected a large number of sensitive documents. Some agents might have a lot of them and others have none. Now they need to distribute the documents throughout the agency such that each person in the LSS has the same amount of data to process.

(i) Assume that there are n agents in the LSS and that there is also a total of n documents. Devise a way for the agents to distribute it: In the end, each agent should have exactly one document. The communication scheme is the same as above.

(ii) How good is your algorithm with respect to time and number of messages? You may assume that arbitrarily many documents can be sent in a single message (i.e., "fax phone call").