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Online and Distributed Algorithms

Exercise Session 5

1. *Asynchronous Blackhole Search with a Sense of Direction:* Consider a scenario where agents have ignorance on the graph topology but possess a sense of direction, an ability (called coding) to decide whether two different paths $\pi_1, \pi_2 \in P[x]$ starting from the same node x also end at the same node, just by looking at the two corresponding sequences of labels $\Lambda(\pi_1)$ and $\Lambda(\pi_2)$, besides the ability (called decoding) to translate from neighbour to neighbour the coded information about paths. For example, a sense of direction exists in a mesh labeled with North, South, East and West (compass labeling) as well as in a hypercube where edges are labeled by the corresponding dimensions (dimension labeling). Formally, an edge-labeled graph (G, λ) has sense of direction if and only if there exists a coding function $c : \mathcal{L} \to \mathcal{N}$ for (G, λ) and a decoding function $d : \mathcal{L} \times \mathcal{N} \to \mathcal{N}$ for c, where $\mathcal{N}$ is a finite set. More precisely, $\forall x, y, z \in V, \forall \pi_1 \in P[x, y], \forall \pi_2 \in P[x, z], c(\Lambda_x(\pi_1)) = c(\Lambda_x(\pi_2)) \Leftrightarrow y = z$, whereas $\forall x \in V, \forall \pi \in P[x]$, with $\Lambda_x(\pi)) = [\alpha_1\alpha_2, \ldots, \alpha_k], d(\alpha_1, c([\alpha_2, \ldots, \alpha_k]) = c([\alpha_1, \alpha_2, \ldots, \alpha_k)$, meaning that with the decoding function it is sufficient to know the first label and the coding of the rest instead of all the labels in order to know the coding of a walk.

   (a) Consider an arbitrary graph with vertices $\{v_0, v_1, \ldots, v_{n-1}\}$ where each edge (v_i, v_j) is labeled at v_i by the label $j - i$. Show that, with this edge labeling, called chordal, every graph can be endowed with a sense of direction. Provide the coding function and its corresponding decoding function. When do two paths from the same node terminate in the same node?

   (b) Other edge labelings may also endow an arbitrary graph with a sense of direction. Show that for an arbitrary graph there is a sense of direction if all edges incident at v_j have the same label l_j at all neighbours of v_j, but a different one at v_j itself, and if all other edges have a label distinct from l_j as well. Provide the coding function and its corresponding decoding function for this edge labeling, called neighboring. When do two paths from the same node terminate in the same node?

   (c) Give an asynchonous $O(n^2)$-time protocol for the blackhole search problem with only 2 agents (instead of $\Delta + 1$ from "Ignorance") in an arbitrary graph G with a sense of direction of known size n but unknown topology.

2. *Asynchronous Blackhole Search:* If the scheduler is asynchronous, prove the claims (a-d).

   (a) It is impossible to determine the exact location of the blackhole if the size n of the graph G is unknown.

(b) It is impossible to verify whether or not there is a blackhole (if agents are in doubt).

(c) It is impossible to have a protocol solving the blackhole problem if there are two or more unexplored nodes.

(d) If G has a cut vertex distinct from the homebase (whose deletion creates two or more components) then it is impossible to determine the exact location of the blackhole.