



# Online and Distributed Algorithms

## Exercise Session 7

1. *Sorting Networks*: Prove or disprove the following claims.

- (a) Given any correct sorting network, adding another comparator at the end destroys the sorting property.
- (b) Given any correct sorting network, adding another comparator at the front does not destroy the sorting property.
- (c) Every correct sorting network needs to have at least one comparator between each two consecutive wires.
- (d) A network of  $n$  wires in which each wire has at least  $\frac{n}{2}$  comparators, being at least one to the previous wire (if any) and at least one to the next wire (if any), is a correct sorting network.
- (e) A network which contains all  $\binom{n}{2}$  comparators between any two of the  $n$  wires, in whatever order they are placed, is a correct sorting network.
- (f) Given any correct sorting network, adding another comparator anywhere does not destroy the sorting property.
- (g) Given any correct sorting network, inverting it (i.e., feeding the input into the output wires and traversing the network from right to left) results in another correct sorting network.
- (h) A network of binary entries and  $n$  wires which consists of two sorting networks of  $\frac{n}{2}$  wires followed by bitonic sorter of  $n$  wires (instead of a merging network) is a correct sorting network.
- (i) A sorting network of  $n > 2$  wires needs at least  $2n$  comparators to be correct.
- (j) A sorting network of  $n > 2$  wires needs at most  $2n$  comparators to be correct.