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Online and Distributed Algorithms

Exercise Session 8

1. *Counting Networks:* The bitonic network is not the only counting network with depth $O(\log^2 n)$. Consider the following definition of a counting network, which consists of a sequence of identical subnetworks: The *Periodic Counting Network*, denoted by PERIOD[$2k$], simply consists of $\log 2k$ BLOCK[$2k$] networks in series, joined so that the i^{th} output wire of one is the i^{th} input wire of the next. But what is a BLOCK[$2k$] network? We start by defining chains and cochains. A level i chain of a sequence $x = x_0, \ldots, x_{n-1}$ *with indices in the binary basis* is a subsequence of x whose indices have the same i low-order binary bits. For instance, the subsequence x^E of entries with even indices is a level 1 chain, and the subsequence x^O of entries with odd indices is a level 0 chain. The A-cochain of x, denoted x_A, is the subsequence whose indices have the two low-order bits 00 or 11, while the B-cochain of x, denoted x_B, is the subsequence whose indices have the two low-order bits 01 or 10. For instance, the sequence $x_0, \ldots, x_7$ has x_0, x_3, x_4, x_7 as A-cochain and x_1, x_2, x_5, x_6 as B-cochain. Given that, a BLOCK[$2k$] network consists of two parallel BLOCK[k] networks, called A-block and B-block, where x^A goes as input to the A-block and x^B goes as input to the B-block, and whose outputs are fed into a EVEN/ODD[2k] network. (Finally, a BLOCK[2] is a single balancer). Draw PERIODIC[8]. Prove that:

   (a) If x and x' are sequences, each having the step property, and pairs x_i and x'_i are routed through a balancer, yielding outputs y_i and y'_i then the sequences y and y' each have the step property.

   (b) Let BLOCK[$2k$] be quiescent with input sequence x and output sequence y. If both x^E and x^O have the step property, so does y.

   (c) Let BLOCK[$2k$] be quiescent with input sequence x and output sequence y. If all level i input chains to a block have the step property, then so do all the level $i-1$ output chains.

   in order to prove that PERIOD[$2k$] is indeed a correct counting network.