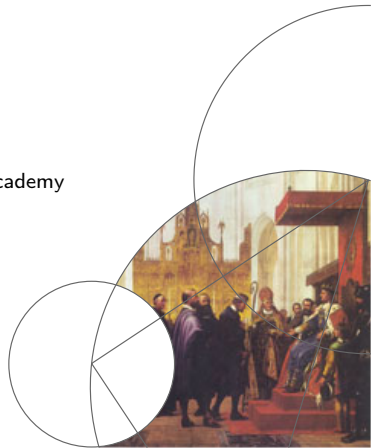




Probing the structure and dynamics of active systems using topological data analysis

Order and Disorder in Random Geometries:
Hyperuniformity and Topological Data Analysis
July 8th, 2026

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Niels Bohr Institute and Niels Bohr International Academy
University of Copenhagen



Introduction



Active matter



A passive system

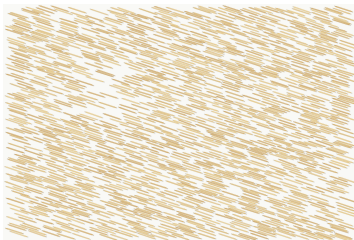


An active system

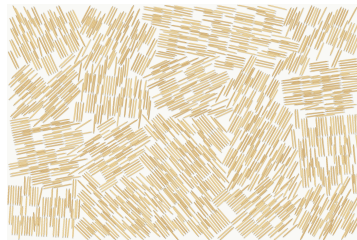
Active systems are characterised by the injection/conversion of energy at the local/small scale.

Such systems are out-of-equilibrium and exhibit several exotic properties such as concerted/collective motion, strange emergent phenomena, and unique phase transitions.

Nematics and nematic order



A very ordered nematic texture



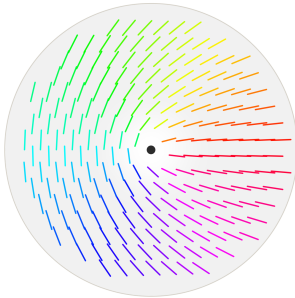
A less ordered nematic texture

We capture the degree of nematic order in the (traceless, symmetric) nematic tensor (here, in 2D):

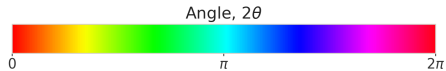
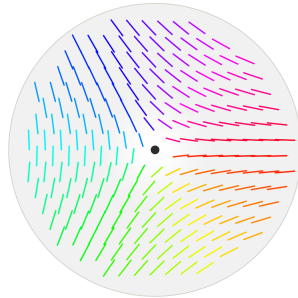
$$\mathbf{Q} = S \left(\mathbf{n} \otimes \mathbf{n} - \frac{\mathbf{I}}{2} \right)$$

Nematic defects

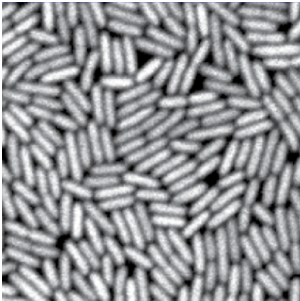
$+\frac{1}{2}$ defect (a "comet")



$-\frac{1}{2}$ defect (a "trefoil")



Active nematics in Nature



Escherichia coli ($25\text{ }\mu\text{m} \times 25\text{ }\mu\text{m}$)



Microtubule-kinesin

Meacock *et al.* (2021), Ardasheva *et al.* (2025)

Active nematics theory

We can model such systems using *active nematohydrodynamics* - PDEs detailing the relationship between $\mathbf{Q}$ and the velocity field of the containing liquid, $\vec{u}$:

$$\left(\partial_t + \vec{u} \cdot \vec{\nabla}\right) \mathbf{Q} - \overset{\text{Co-rotation}}{\mathbf{S}} = -\frac{1}{\gamma} \overset{\text{Molecular field}}{\mathbf{H}}$$

$$\vec{\nabla} \cdot \vec{u} = 0$$

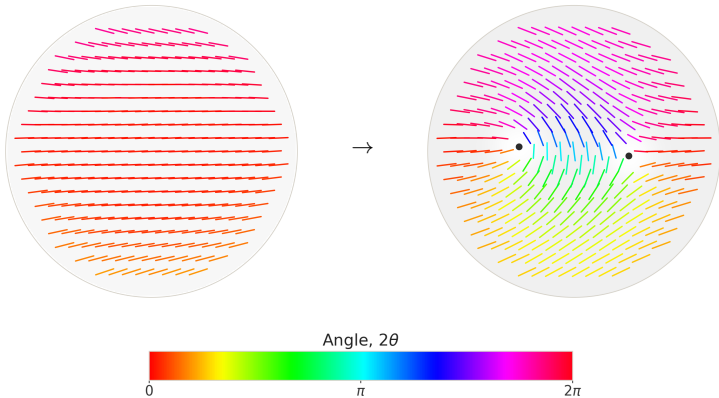
$$\left(\partial_t + \vec{u} \cdot \vec{\nabla}\right) \vec{u} = \vec{\nabla} \cdot \left(\overset{\text{Strain}}{\Pi_{\text{Viscous}}} + \overset{\text{Pressure, elasticity}}{\Pi_{\text{Passive}}} + \Pi_{\text{Active}} \right)$$

so that the active stress: $\Pi_{\text{Active}} = -\zeta \mathbf{Q}$ injects energy into the system.

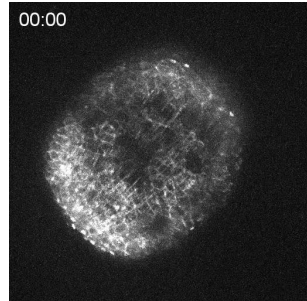
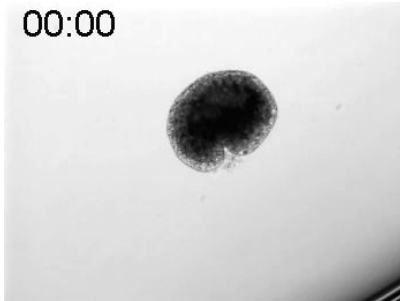


Active nematics and defects

Above a certain level of activity, defects spontaneously form (in pairs):



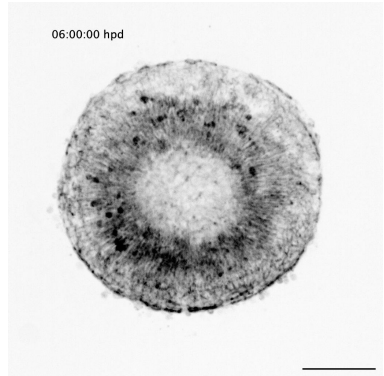
Defects and biology



Hydra, Approx. 10^5 cells, FoV $1114\ \mu\text{m}/600\ \mu\text{m}$

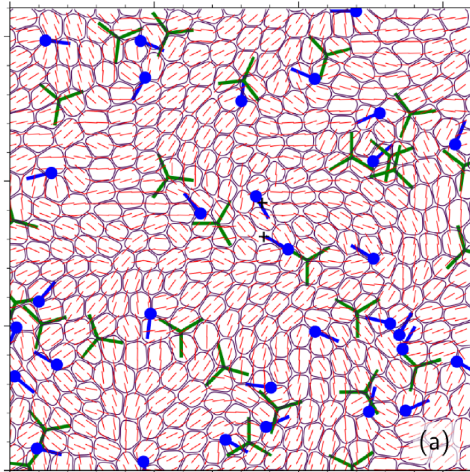
Livshits *et al.* (2017)

Defects and biology



Hydra, Approx. 10^5 cells, Scale bar: 100 μm

Defects and biology



Epithelial cell monolayer (simulation), $+\frac{1}{2}$ and $-\frac{1}{2}$ defects.

Monfared et al. (2023)

Story 1: Defect organisation in active nematics at criticality



Defect organisation in active nematics at criticality

In practice, we study nematohydrodynamical theory via simulations.

- 2048×2048 square mesh domains
- Periodic boundary conditions
- Hybrid Lattice Boltzmann methods
- Simulations burn in until a steady state is reached
- After that we sample according to the correlation time of the fields

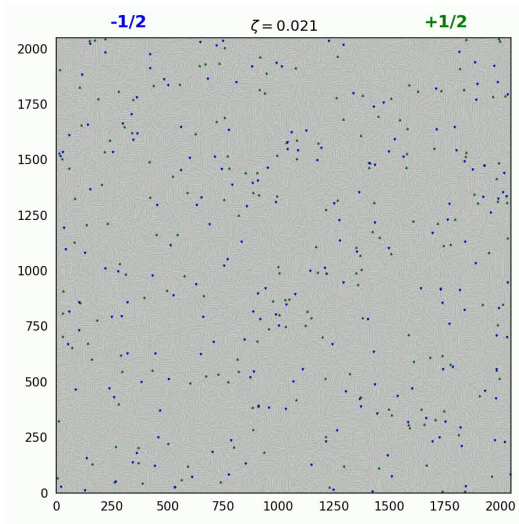


Simon Andersen

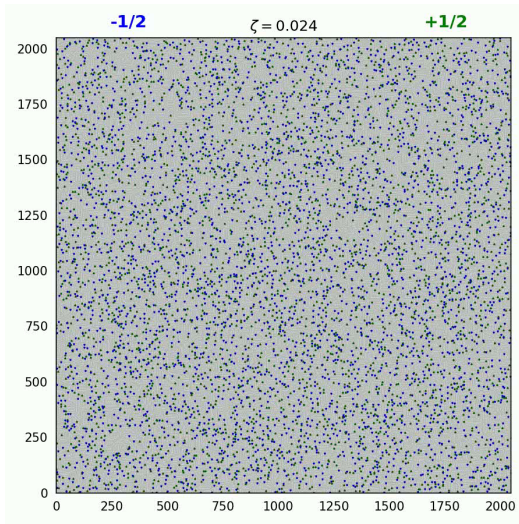
Marenduzzo *et al.* (2007)



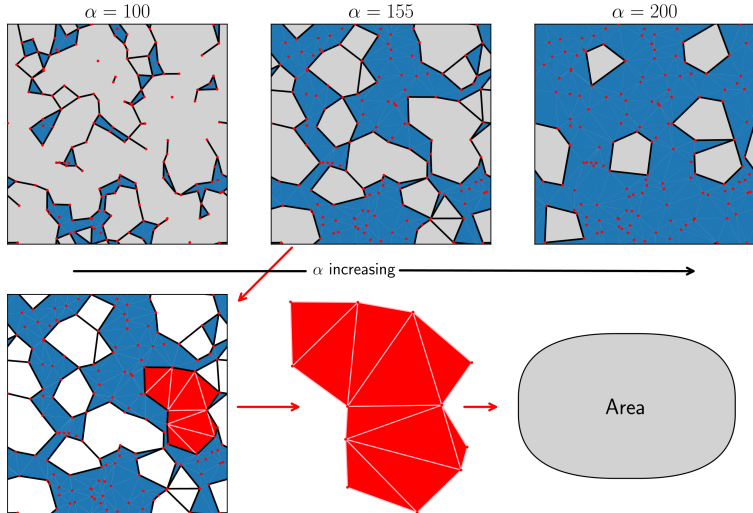
Simulation - below transition



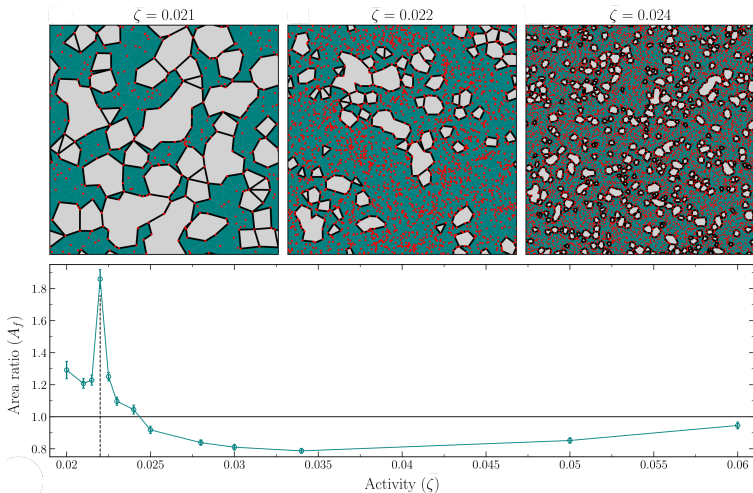
Simulation - above transition



Estimation of the largest defect-free patch

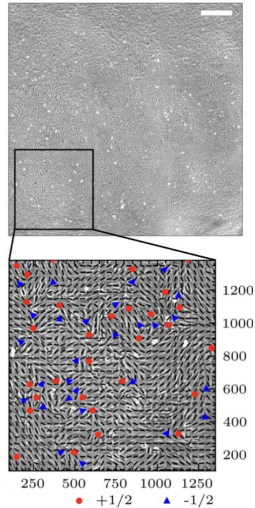


The sizes of defect-free patches fluctuate at criticality



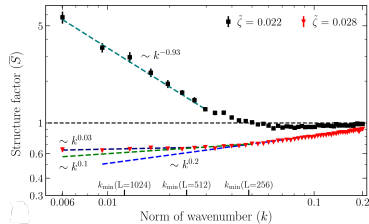
Additional remarks

- The phenomenon is also found in actual data (endothelial cell monolayer, scale bar: 500 μm).



Additional remarks

- The phenomenon is also found in actual data (endothelial cell monolayer, scale bar: 500 μm).
- The defect structures are (effectively):
 - anti-hyperuniform near the transition.
 - uniform above the transition.



Story 2: Phase transitions from vorticity data



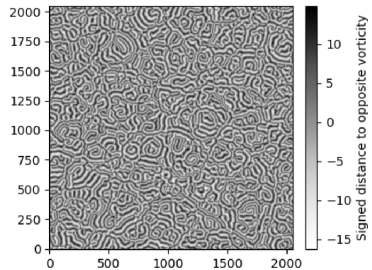
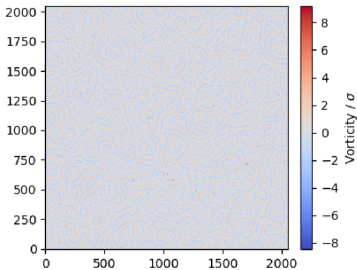
Phase transitions from vorticity data

We turn our attention to the vorticity field in our simulations: $\vec{\omega} = \vec{\nabla} \times \vec{u}$.

And compute the SEDT of the vorticity domains.

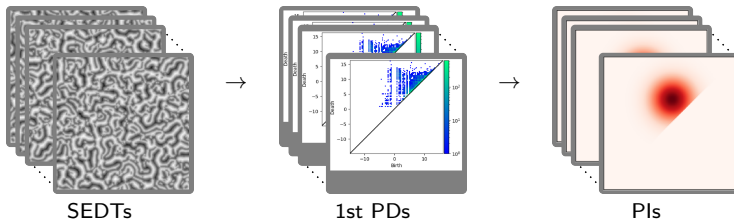


Lasse Bonn



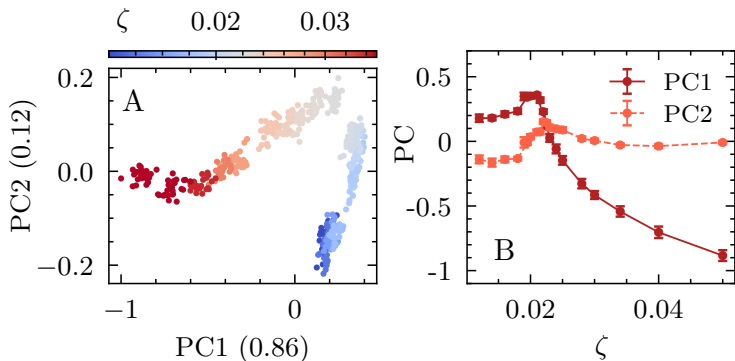
Phase transitions from vorticity data

We compute (sublevel-set) persistent homology from the SEDT of these data, vectorize these to (arctan-weighted Gaussian kernels) persistence images, and submit them to PCA.

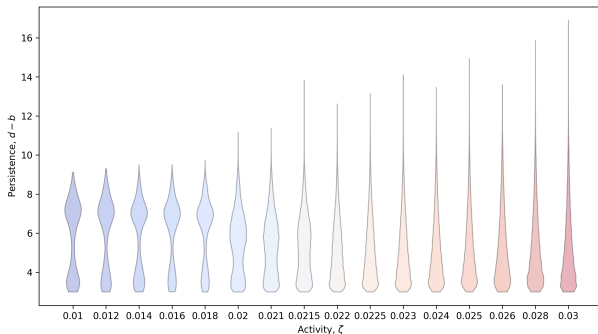


We use 5 selected frames for 15 activities with $\zeta \in [0.05, 0.035]$ repeated for 5 random starting conditions.

PCA identifies the transition



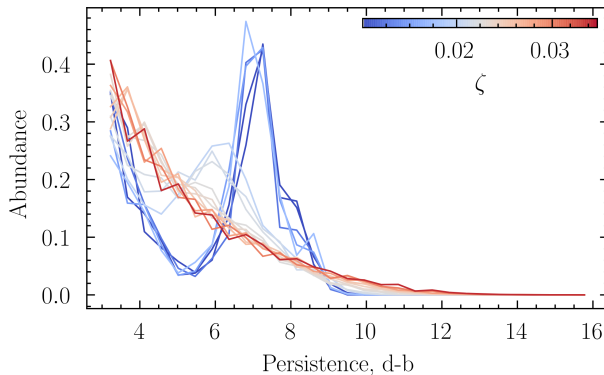
Persistence distributions tells us how



Persistence (lengths) are measured in units of the active length scale: $l_a = \sqrt{\frac{K}{\zeta}}$



Persistence distributions tells us how

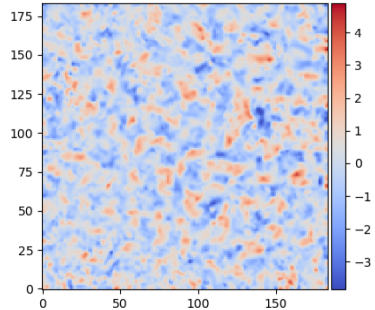


Persistence (lengths) are measured in units of the active length scale: $l_a = \sqrt{\frac{K}{\zeta}}$



Additional remarks

- Vorticity field data are much more abundant than say defect locations (data from time-resolved imaging of microtubules).

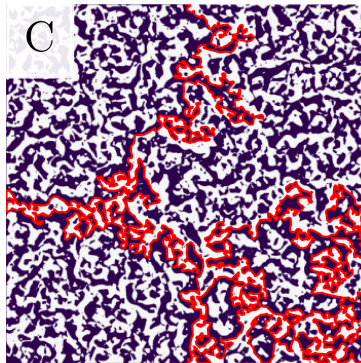


Bantysh (2026), Schramm (2000), Andersen *et al.* (2025)



Additional remarks

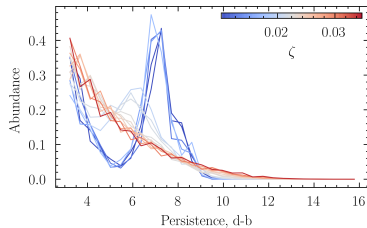
- Vorticity field data are much more abundant than say defect locations (data from time-resolved imaging of microtubules).
- The transition is quite interesting. Above the transition, the zero-vorticity line exhibits SLE₆ implying conformal invariance.



Bantysh (2026), Schramm (2000), Andersen *et al.* (2025)

Additional remarks

- Vorticity field data are much more abundant than say defect locations (data from time-resolved imaging of microtubules).
- The transition is quite interesting. Above the transition, the zero-vorticity line exhibits SLE₆ implying conformal invariance.
- Is this a two-stage transition?



Bantysh (2026), Schramm (2000), Andersen *et al.* (2025)



Story 3: Active particles alter the topology of passive media



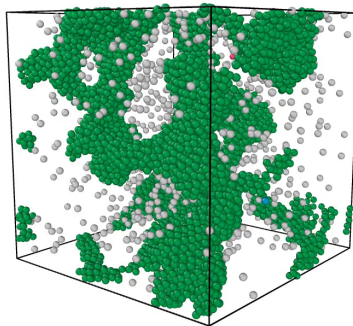
Active particles in passive media

We begin with a passive, colloidal gel
- doped with 10% active

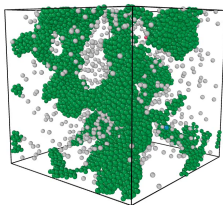
The activity model has two parameters:

- The magnitude of the self-propelling force, F
- A timescale setting the reorientation frequency of force exerted by the active particles, τ

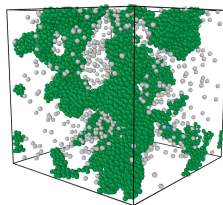
And all particles are subjected to a Morse potential which make them sticky and give them an effective radius, r_c . The gel particles are slightly polydisperse.



Active particles in passive media



$$F/F_p = 0.08 \text{ (Large } \tau \text{)}$$



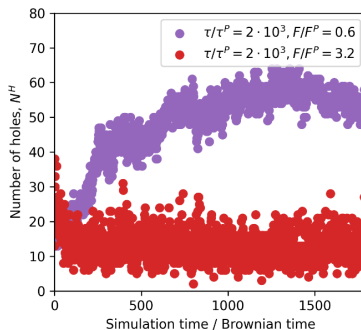
$$F/F_p = 4 \text{ (Large } \tau \text{)}$$

TDA, TDA, TDA

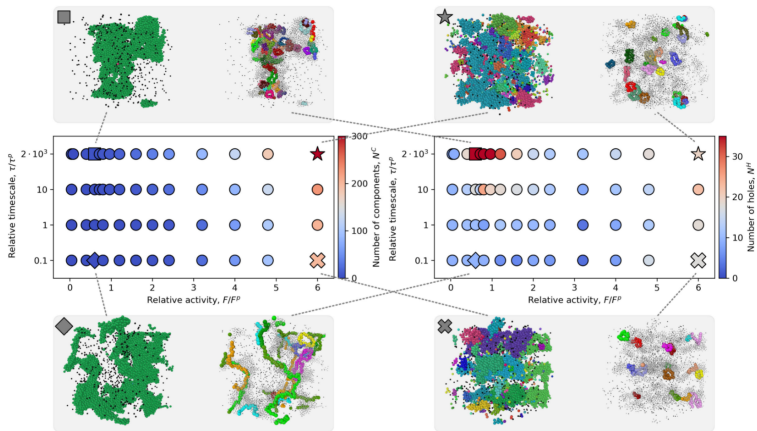
TDA offers an attractive framework for quantifying this.

We compute alpha-shape filtrations from the positions of the *passive* particles and say that:

- Two particles interact(/overlap) if they are closer than $2r_c$
- A 1-feature is a *hole* if another particle can pass through it without interacting with the particles forming the *hole*



A topological Goldilocks zone



Wrapping up



Bibliography

We have looked at results from:

- Active particles knead three-dimensional gels into porous structures
Pedersen *et al.* (2025)
Phys. Rev. Lett. 133(22), 228301
- Anti-hyperuniform critical states of active topological defects
Andersen *et al.* (2025)
Rep. Prog. Phys. 88(10), 108101
- Fully developed active turbulence defined through a non-equilibrium phase transition
Bonn *et al.* (2026)
Nat. Phys., To appear
- Cell Division Sets a Universal Flow Geometry in Cell Layers
Ma *et al.* (2026)
Proc. Nat. Acad. Sci. U. S. A., To appear



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- Wei Feng (Northwest U)
- Xiao Liu (Beihang U)



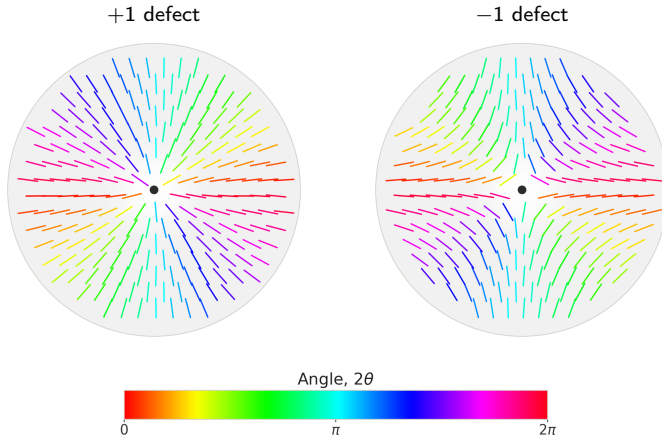
VILLUM FONDEN



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International Academy



More defects



More active nematics theory 1

$$\left(\partial_t + \vec{u} \cdot \vec{\nabla}\right) \mathbf{Q} - \mathbf{S} = -\frac{1}{\gamma} \mathbf{H}$$

where:

$$\mathbf{H} = \left(\frac{\delta \mathcal{F}}{\delta \mathbf{Q}}\right)^{\text{ST}} \quad \mathcal{F} = \int d\vec{r} C(1 - \text{Tr}(\mathbf{Q}^2)) \text{Tr}(\mathbf{Q}^2) + \frac{K}{2} (\vec{\nabla} \cdot \mathbf{Q})^2$$

and:

$$\mathbf{S} = (\lambda \mathbf{E} + \Omega) \cdot (\mathbf{Q} + \mathbf{I}/2) + (\mathbf{Q} + \mathbf{I}/2) \cdot (\lambda \mathbf{E} - \Omega) - 2\lambda (\mathbf{Q} + \mathbf{I}/2) (\mathbf{Q} : \vec{\nabla} \vec{u})$$

$$\Omega = \frac{1}{2} (\vec{\nabla} \vec{u} - (\vec{\nabla} \vec{u})^\top) \quad \mathbf{E} = \frac{1}{2} \left(\vec{\nabla} \vec{u} + (\vec{\nabla} \vec{u})^\top \right)$$



More active nematics theory 2

$$\left(\partial_t + \vec{u} \cdot \vec{\nabla}\right) \vec{u} = \vec{\nabla} \cdot (\Pi_{\text{Viscous}} + \Pi_{\text{Passive}} + \Pi_{\text{Active}})$$

where:

$$\Pi_{\text{Viscous}} = 2\eta \mathbf{E}$$

$$\begin{aligned} \Pi_{\text{Passive}} = & -p\delta_{ij} + 2\lambda(Q_{ij} + \delta_{ij}/2)(Q_{lk}H_{kl}) \\ & -\lambda H_{ik}(Q_{kj} + \delta_{kj}/2) - \lambda(Q_{ik} + \delta_{ik}/2)H_{kj} \\ & -\partial_i Q_{kl} \frac{\delta \mathcal{F}}{\delta \partial_j Q_{lk}} + Q_{ik}H_{kj} - H_{ik}Q_{kj} \end{aligned}$$

$$\Pi_{\text{Active}} = -\zeta \mathbf{Q}$$

and:

$$\vec{\nabla} \cdot \vec{u} = 0$$



Nematohydrodynamical parameters

Parameter	Symbol	Value	Dimension
Bulk constant	C	1	M/T^2
Elastic constant	K	0.05	ML^2/T^2
Flow alignment	λ	1	1
Rotational viscosity	γ	20	M/T
Solvent viscosity	η	100/6	M/T
LB relaxation time	τ_{LB}	1	T
Activity	ζ	-	M/T^2

