Time Series, Random Fields and beyond

Joint German-Japanese Fall School

September 23 - September 27, 2024

Ulm University

HOSTED BY ULM UNIVERSITY

Organizing Institutes:

- The Institute of Statistical Mathematics - Japan
- The University of Tokyo – Japan
- Tohoku University - Japan
- Institute of Mathematical Finance & Institute of Stochastics – Ulm University, Germany

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Content

Monday, September 23th

Time	Agenda
08:30 – 09:00	Registration
09:00 – 09:10	Opening ▪ Evgeny Spodarev - Director of the Institute of Stochastics, Ulm University ▪ Dieter Rautenbach - Vice President Career, Ulm University
09:10 - 10:30	Gennady Samorodnitsky "New issues in extremes: imperfect extremes, extremal clustering in high dimension, causality and privacy in extreme value analysis" (Part 1)
10:30 – 10:50	Break
10:50 – 12:10	Kengo Kamatani "Markov Chain Monte Carlo" (Part 1)
12:10 - 14:00	Lunch Break
14:00 - 15:20	Hermine Biermé "Mean Geometry of 2D Random Field" (Part 1)
15:20 – 15:40	Coffee Break
15:40 – 16:10	Satoshi Kuriki "Minkowski functionals for non-Gaussian orthogonally invariant random fields on spheres"
16:10 - 16:40	Anna Vidotto "The Geometry of Time-Dependent Spherical Random Fields"
16:40 - 17:10	Tomonari Sei "On construction of Markov chains with given dependence and marginal stationary distributions"
17:30	Get Together

Tuesday, September 24th

Time	Agenda
09:00 – 10:20	Gennady Samorodnitsky "New issues in extremes: imperfect extremes, extremal clustering in high dimension, causality and privacy in extreme value analysis" (Part 2)
10:20 – 10:40	Break
10:40 - 12:00	Yasumasa Matsuda "Fourier analysis of surface time series" (Part 1)
12:00 – 14:00	Lunch Break
14:00 – 15:20	Hermine Biermé "Mean Geometry of 2D Random Fields " (Part 2)
15:20 - 15:40	Coffee Break
15:40 - 16:10	Yuichi Goto "Optimal tests for the absence of random individual effects in large "n" and small "T" dynamic panels"
16:10 – 16:40	Matthias Neumann "Stochastic 3D modeling of the nanoporous binder-additive phase in battery electrodes"
16:40 – 17:10	Yuma Uehara "On Whittle estimation for Levy continuous-time moving average process"
17:30	Poster Section/ Get Together

Wednesday, September 25th

Time	Agenda
09:00 – 10:20	Kengo Kamatani "Markov Chain Monte Carlo" (Part 2)
10:20 – 10:40	Break
10:40 - 12:00	Michael Klatt " Hyperuniformity: a hidden long-range order in random geometric systems" (Part 1)
12:00 – 12:05	Group Photo
12:05 – 14:00	Lunch Break
15:30	Guided tour of the Ulm old town and Cathedral – Meeting Point: Tourist-Information "Stadthaus" at Münsterplatz

Thursday, September 26th

Time	Agenda
09:00 – 10:20	Zakhar Kabluchko "Stationary random fields appearing in number theory" (Part 1)
10:20 – 10:40	Break
10:40 - 12:00	Yasumasa Matsuda "Fourier analysis of surface time series" (Part 2)
12:00 – 14:00	Lunch Break
14:00 - 14:30	Giacomo Francisci "Uniform limit theorems for point processes"

Thursday, September 26th

Time	Agenda
14:30 – 15:00	Daisuke Kurisu "Causal Inference for Random Objects"
15:00 – 15:30	Hiroki Masuda „Outlier-Resistant inference“
15:30 - 15:50	Coffee Break
15:50 – 16:20	Teppei Ogihara "Efficient drift parameter estimation for ergodic solutions of backward SDEs"
16:20 - 16:50	Benedikt Prifling "Investigating the aging behavior of anodes in solid-oxide fuel cell using stochastic 3D microstructure modeling based on excursion sets of Gaussian random fields“
16:50 - 17:20	Matthias Weber " A stochastic 3D microstructure model for loam and sand based on Gaussian random fields"
19:00	Conference Dinner at Restaurant "Drei Kannen"

Friday, September 27th

Time	Agenda
09:00 – 10:20	Michael Klatt " Hyperuniformity: a hidden long-range order in random geometric systems" (Part 2)
10:20 – 10:40	Break
10:40 - 12:00	Zakhar Kabluchko "Stationary random fields appearing in number theory" (Part 2)

Abstracts – Lectures & Talks

Monday, September 23th

- **Prof. Dr. Gennady Samorodnitsky, Cornell University (USA)**

New issues in extremes: imperfect extremes, extremal clustering in high dimension, causality and privacy in extreme value analysis

Modern applications of extreme value analysis require going beyond analyzing extremes of an i.i.d. sequence or extremes of a weakly dependent stationary process or a random field in 2 or 3 dimensions. The extremes may be truncated and some of them may be simply missing. We will learn that, in some cases, one can still obtain useful information about the extremes. A particularly difficult case of the curse of dimensionality occurs when one needs to analyze extremes in a high dimension. This often requires estimation of a measure on a high-dimensional sphere, and this must be done based on a relatively small number of imperfect extremes. Finding low-dimensional structures in the support of this measure is crucial, and we will learn some techniques for doing so, including the spectral clustering of the extremes and the kernel PCA approach. In many cases statistical analysis of non-public data requires public results of the analysis to be privatized in the sense on revealing certain personal information contained in the data. The usual approaches for achieving that appear to be unsuitable in the extreme value analysis. We will learn what techniques can be used to achieve privacy when working with extremes. Causal statistics aims to detect causal relations between different random objects. Doing so in extreme value analysis is both important and difficult. We will learn some new techniques for causal extreme value analysis.

- **Prof. Dr. Kengo Kamatani, Institute of Statistical Mathematics (Japan)**

Markov Chain Monte Carlo

In my presentation, I'll start with a brief overview of Markov chains, highlighting their mathematical foundations, coupling mechanisms, and some results on convergence. Following this, I'll explore Markov Chain Monte Carlo methods, covering essential algorithms. Lastly, I'll briefly discuss some recent progress in the area of non-reversible chains.

- **Prof. Dr. Hermine Biermé, University of Tours (France)**

Mean Geometry of 2D Random Fields

In these lectures I will mainly present joint works with Ag-nès Desolneux (CNRS, ENS Paris Saclay) in which we consider three geometric characteristics of stationary 2D random field excursion sets, called Lipschitz-Killing curvatures, which are linked to the area, perimeter and Euler characteristic of these sets. By adopting a weak functional framework, we obtain explicit formulas for their mean values which make it possible to extend known results in the context of smooth Gaussian surfaces to non-Gaussian shot-noise fields. In particular this framework allows to recover results from stochastic geometry on boolean models and to shed new lights on the so-called Gaussian kinematic formula of Robert Adler and Jonathan Taylor.

- **Prof. Dr. Satoshi Kuriki, Institute of Statistical Mathematics (Japan)**

Minkowski functionals for non-Gaussian orthogonally invariant random fields on spheres

The Minkowski functionals are statistics for capturing morphological features, typically in 2 or 3-dimensional images. In particular, in astrophysics, it has been used to gasp non-Gaussian information in the large-scale structure of the Universe, the CMB, and so on. With such applications in mind, in our previous work, we have obtained the asymptotic expansion formulas for the expected Minkowski functionals for weakly non-Gaussian isotropic random fields (i.e., central limit random fields) in arbitrary dimensional Euclidian space. In this talk, we consider non-Gaussian orthogonally invariant random fields on spheres. The CMB is regarded as a restriction of a 3-dimensional isotropic random field to a 2-dimensional sphere, hence orthogonally invariant. We present the expected Minkowski functional formula for non-Gaussian random fields including Gaussian, Chi-square, and weakly non-Gaussian random fields. We also discuss how restrictive an orthogonally invariant random field is if it is defined as a restriction to an isotropic random field on higher dimensional Euclidean space."

- **Dr.Anna Vidotto, University of Naples Federico II (Italy)**

The Geometry of Time-Dependent Spherical Random Fields

In this talk, we consider fluctuations over time for the area of the excursion sets and the length of level curves of time-dependent Gaussian spherical random fields. We focus on both long and short memory assumptions; in the former case, we show that the fluctuations are dominated by a single Wiener chaotic component and the existence of cancellation levels where the variance is asymptotically of smaller order. In the short memory case, we show that all Wiener chaoses contribute in the limit, no cancellation occurs and a Central Limit Theorem can be established by Breuer-Major type arguments. The talk is based on two articles written together with Domenico Marinucci and Maurizia Rossi.

- **Prof. Dr.Tomonari Sei, The University of Tokyo (Japan)**

On construction of Markov chains with given dependence and marginal stationary distributions
A method of constructing Markov chains on finite state spaces is provided. The chain is specified by three constraints: stationarity, dependence and marginal distributions. The generalized Pythagorean theorem in information geometry plays a central role in the construction. Integer-valued autoregressive processes are considered for illustration.

Tuesday, September 24th

- **Prof. Dr. Gennady Samorodnitsky, Cornell University (USA)**

New issues in extremes: imperfect extremes, extremal clustering in high dimension, causality and privacy in extreme value analysis (Part 2).

- **Prof. Dr. Yasumasa Matsuda, Tohoku University (Japan)**

Fourier analysis of surface time series:
Fourier analysis has been successfully applied in time series analysis with popular tools of discrete Fourier transform, periodogram, spectral density function that led to fruitful theoretical and empirical applications in non-parametric or parametric inference. We have interests in extending Fourier analysis of time series to that of spatial/spatio-temporal data, or surface time series in recent terminology. We extend a discrete Fourier transform and periodogram of time series to those for spatial data and examine conditions under which they have good asymptotic properties held in time series cases, i.e. asymptotic independence. As an application of them, we show theories and practices of spatial CARMA model estimation by Whittle likelihood. Finally, we extend Fourier analysis of spatial data to that of surface time series as the goal of this talk. We will introduce functional principal component analysis as a good alternative to Fourier analysis of surface time series, by which we can clarify interesting features regarded as advantages and disadvantages of Fourier analysis. Specifically, we will discuss the extensions of Fourier analysis from time series to surface time series in the following schedule: ° Review of Fourier analysis of time series. °Spatial Continuous Autoregressive and Moving Average (CARMA) models. °Fourier analysis of spatial data with applications to CARMA model estimation. °Fourier analysis of surface time series. °Functional principal component analysis (fPCA) to surface time series. °Empirical applications

- **Dr. Yuichi Goto, Kyushu University, Faculty of Mathematics (Japan)**

Optimal tests for the absence of random individual effects in large "n" and small "T" dynamic panels

Yuichi Goto[1], Nezar Bennala[2], Marc Hallin[2]
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In this paper, we consider the problem of testing the absence of individual effects in dynamic $n \times T$ panels with AR(p) disturbances. We establish a local asymptotic normality property (LAN) for n (the number of individuals) tending to infinity and T (the time series length) fixed. Then, we construct locally asymptotically optimal parametric and pseudo-Gaussian tests.

- **Dr. Matthias Neumann, Graz University of Technology (Austria)**

Stochastic 3D modeling of the nanoporous binder-additive phase in battery electrodes

A stochastic 3D model for the nanoporous binder-additive phase in hierarchically structured electrodes of lithium-ion batteries is presented. The considered binder-additive phase consists of carbon black, polyvinylidene difluoride binder and graphite particles. For stochastic 3D mod-eling, we use a three-step approach combining excursion sets of Gaussian random fields with germ-grain models. First, the graphite particles extracted from image data are modeled by a Boolean model with ellipsoidal grains. Second, the union of carbon black and binder is modeled by an excursion set of a Gaussian random field in the complement of the graphite particles. Third, large pore regions within the union of carbon black and binder are modeled by a Boolean model with spherical grains. The model is calibrated to tomographic image data of cathodes in lithium-ion batteries acquired by focused ion beam scanning electron microscopy. Subsequently, model validation is performed by comparing various morphological descriptors, that are not used for model fitting, of both model realizations and measured image data. Moreover, we use the validated model for generating virtual, yet realistic, image data of the nanoporous binder-additive phase with systematic variations in the volume fraction of graphite particles. The latter can be controlled by adjusting the intensity of the Boolean model with ellipsoidal grains. Based on this virtual analysis, we quantitatively study the influence of graphite particles on morphological descriptors as well as on effective transport properties such as effective conductivity in the binder-additive phase and effective diffusivity in the pore space.

- **Dr. Yuma Uehara, Kansai University (Japan)**

On Whittle estimation for Levy continuous-time moving average process

In this talk, we consider the estimation problem of Levy continuous-time moving average process with memory property. We utilize Whittle likelihood function in order to estimate the parameter in the kernel. We present the asymptotic behavior of the estimator and related numerical result.

Wednesday, September 25th

- **Prof. Dr. Kengo Kamatani, Institute of Statistical Mathematics (Japan)**

Markov Chain Monte Carlo (Part 2).

- **Dr. Michael Klatt, German Aerospace Center (DLR), (Germany)**

Hyperuniformity: a hidden long-range order in random geometric systems
A random geometric system can be both locally similar to complete spatial randomness and globally homogeneous like a lattice. Such an anomalous suppression of large-scale density fluctuations is known as hyperuniformity, and it has profound implications for the mathematical and physical properties of the system. In these two lectures, we will take a closer look at the definitions and properties of hyperuniform random geometric systems, highlighting some of the subtleties of this fascinating field of research. We will discuss prominent examples as well as physical implications and relations to different fields of mathematics. Moreover, we will touch upon the connections to other types of novel short- and long-range order, like rigidity or quasicrystal symmetries.

Thursday, September 26th

- **Prof. Dr. Zakhar Kabluchko, Uni Münster, (Germany)**

Stationary random fields appearing in number theory

These lectures are devoted to stationary random fields appearing in analytic number theory. Here is a typical example. Consider a function $f : \mathbb{Z} \to \{0, 1\}$ defined by $f(x) = 1$ if x is a square-free number, and $f(x) = 0$ otherwise. Let U_n be a random variable following a uniform distribution on the finite set $\{1, 2, \dots, n\}$. Consider a random process $X_n(t) := f(U_n + t)$, $t \in \mathbb{Z}$. It can be shown that, as $n \to \infty$, the process $(X_n(t))_{t\in\mathbb{Z}}$ converges weakly to a certain stationary, $\{0, 1\}$-valued stochastic process $(X(t))_{t\in\mathbb{Z}}$. In some sense, the limiting process $X(t)$ describes the statistics of square-free numbers near a "uniform random integer". For example, the probability of the event $\{X(0) = 1\}$ equals $6/\pi^2$, which corresponds to a classical fact that the asymptotic density of square-free integers is $6/\pi^2$.

There are many other stationary random processes and fields of this type appearing quite naturally in number theory. Typically, such fields have purely discrete spectral measure (a property that is quite unusual for probabilists) and zero entropy. Also, these fields are weakly mixing (hence, ergodic) but not mixing. In these lectures we will review results obtained by several authors on random fields of this type. The lectures will rely only on elementary facts from number theory.

- **Prof. Dr. Yasumasa Matsuda, Tohoku University (Japan)**

Fourier analysis of surface time series (Part 2)

- **Dr. Giacomo Francisci, Ulm University (Germany)**

Uniform limit theorems for point processes
We study the asymptotic behavior of a sequence of i.i.d. point processes over a class of bounded measurable functions. Specifically, under suitable measurability and moment assumptions, we provide sufficient conditions for the uniform law of large numbers (LLN) and the uniform central limit the-Orem in terms of random metric entropy. Additionally, we establish the uniform rates of convergence for the LLN. These results directly apply to sequences of tree-indexed random variables, which may be dependent if they share a common parent vertex. As a consequence, we obtain uniform consistency and uniform asymptotic normality for Lotka-Nagaev and Harris-type estimators for functionals of tree-indexed random variables. Furthermore, we derive the uniform asymptotic properties of half space depth allowing an in-quiry concerning medians and quantiles of tree-indexed multivariate random variables.

- **Dr. Daisuke Kurisu, The University of Tokyo (Japan)**

Causal Inference for Random Objects
Adjusting for confounding and imbalance when establishing statistical relationships is an increasingly important task, and causal inference methods have emerged as the most popular tool to achieve this. Causal inference has been developed mainly for scalar outcomes and recently for distributional outcomes. We introduce here a general framework for causal inference when outcomes reside in general geodesic metric spaces, where we draw on a novel geodesic calculus that facilitates scalar multiplication for geodesics and the characterization of treatment effects through the concept of the geodesic average treatment effect. Using ideas from Frechet regression, we develop estimation methods of the geodesic average treatment effect and derive consistency and rates of convergence for the proposed estimators. We also study uncertainty quantification and inference for the treatment effect. Our methodology is illustrated by a simulation study and real data examples for compositional outcomes of U.S. statewise energy source data to study the effect of coal mining, network data of New York taxi trips, where the effect of the COVID-19 pandemic is of interest, and brain functional connectivity network data to study the effect of Alzheimer's disease.

- **Prof. Dr. Hiroki Masuda, The University of Tokyo (Japan)**

Outlier-resistant inference without jump-detection filter
We propose a modified Gaussian quasi-likelihood through density-power weighting. It is designed for estimating the continuous part of a stochastic process in the presence of jump-type and/or spike-type outliers. We will present the asymptotic distributional properties of the proposed estimator, followed by illustrative simulation results which show how much the estimator is robust against outliers and is sensitive to (one-parameter) fine-tuning.

- **Dr. Teppei Ogihara, The University of Tokyo (Japan)**

Efficient drift parameter estimation for ergodic solutions of backward SDEs
This study addresses the problem of asymptotically efficient estimation of drift parameters in ergodic backward stochastic differential equations (BSDEs). Unlike previous studies that assumed known or parametric models for the diffusion coefficient, we consider a more general setting where the diffusion coefficient is unknown and nonparametric. We propose a maximum likelihood-type estimator based on discrete observation data and prove its consistency and asymptotic normality. Furthermore, we demonstrate that the proposed method achieves the optimal asymptotic variance attainable when the diffusion coefficient is observable. This enables efficient parameter estimation even in models with unknown stochastic volatility. To complement the theoretical results, we also conducted numerical experiments to confirm the effectiveness of the proposed method. The findings of this research are expected to contribute to various fields where BSDEs are applied, such as the analysis of stochastic volatility models in financial engineering and optimal control problems.

- **Dr. Benedikt Prifling, Ulm University (Germany)**

Investigating the aging behavior of anodes in solid-oxide fuel cell using stochastic 3D microstructure modeling based on excursion sets of Gaussian random fields
Solid oxide fuel cells (SOFCs) are becoming increasingly important due to their high electrical efficiency, the flexible choice of fuels and relatively low emissions of pollutants. However, the increasingly growing demands for electrochemical devices require further performance improvements as for example by reducing degradation effects. Since it is well known that the 3D structure of the electrodes has a profound impact on the resulting performance, a deeper understanding of process-structure-property relationships is required. Since tomographic imaging is time-consuming and cost-intensive, virtual materials testing is a promising approach to tackle this issue by generating a large number of virtual, but realistic microstructures drawn from a stochastic 3D microstructure model. In this talk, we present the calibration of such a model to image data, which is segmented

into gadolinium-doped ceria (GDC), nickel and pore space [1]. More precisely, the pluri-Gaussian model described in [2] is calibrated to three-phase microstructures obtained by a numerical aging simulation [3]. Since one and the same model type is used for each aging scenario, interpolation within the parameter space allows for predicting the 3D microstructure of SOFC anodes for aging conditions, for which no tomographic image data is available.

[1] S. Weber, B. Prifling, M. Juckel, Y. Liu, M. Wieler, A. Weber, N. Menzler,V.Schmidt, Comparing the 3D morphology of solid-oxide fuel cell anodes for different manufacturing processes, aging durations, and operating temperatures.
[2] M. Neumann, M. Osenberg, A. Hilger, D. Franzen, T. Turek, I. Manke, V.Schmidt, On a pluri-Gaussian model for three-phase microstructures, with applications to 3D image data of gas-diffusion electrodes. Computational Materials Science 156 (2019),325-331.
[3] S. Weber, B. Prifling, R. K. Jeela, D. Schneider, B. Nestler, V. Schmidt, Quantitative investigating the aging behavior of anodes in solid-oxide fuel cells by combining stochastic 3D microstructure modeling with physics-based aging simulations. Working paper (under preparation)

- **Dr. Matthias Weber, Ulm University (Germany)**

A stochastic 3D microstructure model for loam and sand based on Gaussian random fields

The 3D microstructure of soils crucially influences the growth of plants and, especially, the organization of plant roots. Nutrient availability and uptake are key factors of plant growth which are specifically linked to the pore space morphology of soil. More precisely, diffusive processes within the pore space need to be further investigated to better understand the relationship between soil morphology and plant root organization. As experimental investigation of diffusion is costly and time-consuming, numerical methods for simulating transport have emerged. Numerical simulations on tomographic image data combined with the assessment of geometrical descriptors have been used to investigate the relationship between pore space morphology and soil gas diffusion [1]. However, different types of soil are extremely heterogeneous and thus, considering only limited amounts of measured image data may not be sufficient for a thorough investigation. In this talk, we present a stochastic 3D microstructure model for different types of soil which is based on excursion sets of Gaussian random fields [2]. Using this model, we obtain a wide variety of simulated, yet realistic soil structures which form the basis for further investigations.

[1] B. Prifling, M. Weber, N. Ray, A. Prechtel, M. Phalempin, S. Schlüter, D. Vetterlein, V. Schmidt, Quantifying the impact of 3D pore space morphology on soil gas diffusion in loam and sand. Transport in Porous Media 149 (2023), 501-527.
[2] M. Weber, B. Prifling, N. Ray, A. Prechtel, M. Phalempin, S. Schlüter, D. Vetterlein, V. Schmidt, Investigating relationships between 3D pore space morphology and soil gas diffusion based on data-driven spatial stochastic modeling. Working paper (under preparation).

Friday, September 27th

- **Dr. Michael Klatt, German Aerospace Center (DLR), (Germany)**

Hyperuniformity a hidden long-range order in random geometric systems (Part 2)

- **Prof. Dr. Zakhar Kabluchko, Münster University (Germany)**

Stationary random fields appearing in number theory (Part 2)

Posters – Title and Abstracts

- **Felix Benning, University Mannheim (Germany)**

Optimization of high-dimensional random functions
Classical worst-case optimization theory neither explains the success of optimization in machine learning, nor helps with step-size selection. We discuss how theory on optimization of high dimensional random functions (a.k.a. random fields or stochastic processes) explains the reliable success of gradient based optimizers in machine learning and present a statistically motived gradient based optimizer (Random Function Descent - RFD) which selects its step sizes based on statistical covariance estimation of the random function.

- **Dr. David Berger, TU Dresden (Germany)**

Divisibility properties of probability measures
The set of infinitely divisible distributions in the space of all probability measures is a well-studied object in stochastics. On this poster we discuss the divisibility property of quasi-infinitely divisible distribution and prove that not every quasi-infinitely divisible distribution is divisible even in the set of (regular) bounded complex-valued measures. Furthermore, we extend the notion of quasi-infinitely divisible distributions by extending the abelian monoid of probability measures to the Grothendieck group and show that the constructed group is torsion-free and divisible.

- **Artur Bille, Ulm University (Germany)**

Random eigenvalues of nanotubes

The hexagonal lattice and its dual, the triangular lattice, serve as powerful models for comprehending the atomic and ring connectivity, respectively, in *graphene* and *carbon* (p, q)*–nanotubes*. The chemical and physical attributes of these two carbon allotropes are closely linked to the average number of closed paths of different lengths $k \in \mathbb{N}_0$ on their respective graph representations. Considering that a carbon (p, q)–nanotube can be thought of as a graphene sheet rolled up in a matter determined by the *chiral vector* (p, q), our findings build upon the study of *random eigenvalues* of the hexagonal and triangular lattices presented in [1]. We show that for any given *chiral vector* (p, q), the sequence of counts of closed paths, expressed in [3] as a triple integral, forms a moment sequence derived from a functional of two independent uniform distributions. Explicit formulas for key characteristics of these distributions, including the probability density function and moment generating function, are presented for specific choices of the chiral vector. Moreover, we demonstrate that as the *circumference* of a (p, q)–nanotube approaches infinity (i.e. $p + q \to \infty$), the (p, q)–nanotube converges to the hexagonal lattice with respect to the number of closed paths for any given length k, indicating the weak convergence of the underlying distributions. All presented results and their corresponding proofs are detailed in [2].

References

[1] A. Bille, V. Buchstaber, S. Coste, S. Kuriki, and E. Spodarev. Random eigenvalues of graphenes and the triangulation of plane, 2023, Arxiv Preprint No 2306.01462, Submitted.

[2] A. Bille, V. Buchstaber, P. Ievlev, S. Novikov, and E. Spodarev. Random eigenvalues of nanotubes, 2024, Preprint.

[3] N. Cotfas. Random walks on carbon nanotubes and quasicrystals. *Journal of Physics A*, 33:2917–2927, 2000.

- **Manuel Hasenbichler, University of Technology Graz, (Austria)**

A classification approach to multivariate counting processes modelling

Suppose, a financial institution wants to infer dependency between different types of loss events. Classically, the marginal losses within some period of time (e.g. per year) and their dependence are modelled separately, using copula theory. But what if small loss events tend to herald larger, more threatening events ("downward spiral")? If a firm adopts a "static" approach and is on such an unfavorable path, it may become insolvent before the end of the year - not necessarily because of a lack of assets, but rather because of a lack of liquid funds.

The numbers of types of loss events that have occurred up to a point in time can be understood as a multivariate counting process, $(N^1, \ldots, N^m)$. The process counting the total number of events then is $N := \sum_{i=1}^m N^i$ and w.l.o.g. we assume that $(N^1, \ldots, N^m)$ have distinct jump times. With the static approach in mind, we propose to model N first and to assign jumps of N their "jump origin" $\{1, \ldots, m\}$ afterwards: Hereby, N is thinned to $(N^1, \ldots, N^m)$ by applying appropriate thinning probabilities - i.e. predictable (!) processes $(p^1, \ldots, p^m)$ with $\sum_{i=1}^m p^i = 1$ almost surely. These are generally unknown, although it can be shown that such processes already exist under mild assumptions. We therefore fix a memory lag $L \in \mathbb{N}$ and model p^i as a measurable map depending on i, the last L jump origins and the last L inter-jump periods. If the latter are re-scaled appropriately, we can Markovianise the problem and, provided that the measurable map is from a suitable parameterised family, show that a cross-entropy minimiser is asymptotically consistent.

This presentation is based on ongoing work.

- **Ly Viet Hoang, Ulm University (Germany)**

Non-ergodic statistics and spectral density estimation for stationary real harmonizable symmetric α-stable processes

We consider a non-ergodic class of stationary real harmonizable symmetric α-stable processes $X = \{X(t) : t \in \mathbb{R}\}$ with a finite symmetric and absolutely continuous control measure. We refer to its density function as the spectral density of X. These processes admit a LePage series representation and are conditionally Gaussian, which allows us to derive the non-ergodic limit of sample functions on X. In particular, we give an explicit expression for the non-ergodic limits of the empirical characteristic function of X and the lag process $\{X(t+h) - X(t) : t \in \mathbb{R}\}$ with $h > 0$, respectively. The process admits an equivalent representation as a series of sinusoidal waves with random frequencies which are i.i.d. with the (normalized) spectral density of X as their probability density function. Based on strongly consistent frequency estimation using the periodogram we present a strongly consistent estimator of the spectral density. The computation of the periodogram is fast and efficient, and our method is not affected by the non-ergodicity of X.

- **Rei Iwafuchi, Tohoku University (Japan)**

High-dimensional Financial Network Estimation with GARCH-based Neural Network

We propose a novel model for estimating high-dimensional financial networks. Few methods have been proposed for estimating large-scale networks involving hundreds of assets. By incorporating spatiotemporal dependencies and combining GARCH models with deep learning, we successfully estimated networks that allow for natural interpretation. Furthermore, the model demonstrated excellent performance as a volatility model. This study is expected to provide a powerful approach for constructing large-scale portfolios and assessing risks by estimating interpretable networks from large-scale financial data.

- **Dr. Michael Juhos, Passau University (Germany)**

The Maclaurin inequality through the probabilistic lens

Here we take a probabilistic look at Maclaurin's inequality, which is a refinement of the classical AM–GM mean inequality. In a natural randomized setting, we obtain limit theorems and show that a reverse inequality holds with high probability. By its definition, Maclaurin's inequality is naturally related to U-statistics. More precisely, given $x_1, \dots, x_n, p \in (0, \infty)$ and $k \in \mathbb{N}$ with $k \le n$, let us define the quantity

$$S_{k,p}^{(n)} = \left(\binom{n}{k}^{-1} \sum_{1 \le i_1 < \cdots < i_k \le n} x_{i_1}^p \cdots x_{i_k}^p \right)^{1/(kp)}.$$

Then as a consequence of the classical Maclaurin inequalities, we know that $S_{k_1,p}^{(n)} \ge S_{k_2,p}^{(n)}$ for $k_1 < k_2$. In the present article we consider the ratio

$$\mathcal{R}_{k_1,k_2,p}^{(n)} := \frac{S_{k_2,p}^{(n)}}{S_{k_1,p}^{(n)}},$$

evaluated at a random vector $(X_1, \dots, X_n)$ sampled either from the normalized surface measure on the ℓ_p^n-sphere or from a distribution generalizing both the uniform distribution on the ℓ_p^n-ball and the cone measure on the ℓ_p^n-sphere; by Maclaurin's inequality, we always have $\mathcal{R}_{k_1,k_2,p}^{(n)} \leq 1$. We derive central limit theorems for $\mathcal{R}_{k_1,k_2,p}^{(n)}$ and $\mathcal{R}_{k_1,n,p}^{(n)}$ as well as Berry–Esseen bounds and a moderate deviations principle for $\mathcal{R}_{k_1,n,p}^{(n)}$, keeping k_1, k_2 fixed, in order to quantify the set of points where $\mathcal{R}_{k_1,k_2,p}^{(n)} > c$ for $c \in (0,1)$, i.e., where Maclaurin's inequality is reversed up to a factor. Our work partly generalizes results concerning the AM–GM inequality obtained by Kabluchko, Prochno, and Vysotsky (2020), Thäle (2021), and Kaufmann and Thäle (2024+).

- **Luca Lotz, German Aerospace Center (DLR), (Germany)**

Stable Matching of Point Processes and Hyperuniformity

Hyperuniformity is a global property of stationary point processes. The variance of the number of points of such processes within an observation window grows substantially more slowly than the volume of the observation window. At a macroscopic level, this results in rather uniformly distributed points. This distinguishes hyperuniform point processes from, for example, Poisson processes, where points are not correlated, and the variance is proportional to the volume of the observation window. The poster explores the construction of hyperuniform thinnings of a broad class of α -mixing stationary point processes. For a small thinning parameter, the thinned point process is locally hardly distinguishable from the original point process. The construction is based on stable matching, which describes a specific pairwise matching of points from two point processes, favoring points in close proximity. The concept is known from the "stable marriage problem", which in this context is uniquely solvable by a simple algorithm. Here the target point process is stably matched with a hyperuniform point process of lower intensity. Upon disregarding points of the target process that lack a matching partner, the resulting thinning inherits hyperuniformity. This work extends a prior result in [1], where a more limited class of point processes was considered.

[1] Klatt, M., Last, G., and Yogeshwaran, D. (2018). Hyperuniform and Rigid Stable Matchings.

- **Dr. Kazuki Nakajima, The Graduate University for Advanced Studies (Japan)**

Extreme value theory for stationary random field on Z^d

Stehr and Rønn-Nielsen (2021) studies a stationary random field on $\mathbb{Z}^d$ and obtain that under some conditions the normalized version of the distribution of the maximum of the field over an increasing sequence of index sets converges to an extreme value distribution. They assume conditions on the increase rate of index sets. We try relaxing the conditions, and, in addition, investigate the cases of faster increase rates.

- **Tran Duc Nguyen, Ulm University (Germany)**

Statistical pre-detection of cracks in 3D concrete images

In practical applications, effectively segmenting cracks in large-scale computed tomography (CT) images holds significant importance for understanding the structural integrity of materials. However, classical methods and Machine Learning algorithms often incur high computational costs when dealing with the substantial size of input images. Hence, a robust algorithm is needed to pre-detect crack regions, enabling focused analysis and reducing computational overhead. The proposed approach addresses this challenge by offering a streamlined method for identifying crack re- gions in CT images with high probability. By efficiently identifying areas of interest, our algorithm allows for a more focused examination of potential anomalies within the material structure. Through comprehensive testing on both semi-synthetic and real 3D CT images, we validate the efficiency of our approach in enhancing crack segmentation while reducing computational resource requirements.

- **Nicolai Palm, LMU, Munich Center for Machine Learning, (Germany)**

An Online Bootstrap for Time Series

Resampling methods such as the bootstrap have proven invaluable in the field of machine learning. However, the applicability of traditional bootstrap methods is limited when dealing with large streams of dependent data, such as time series or spatially correlated observations. In this paper, we propose a novel bootstrap method that is designed to account for data dependencies and can be executed online, making it particularly suitable for real-time applications. This method is based on an autoregressive sequence of increasingly dependent resampling weights. We prove the theoretical validity of the proposed bootstrap scheme under general conditions. We demonstrate the effectiveness of our approach through extensive simulations and show that it provides reliable uncertainty quantification even in the presence of complex data dependencies. Our work bridges the gap between classical resampling techniques and the demands of modern data analysis, providing a valuable tool for researchers and practitioners in dynamic, data-rich environments.

- **Francesca Pistolato, University of Luxembourg (Luxembourg)**

Lipschitz-Killing curvatures for excursion sets of spin random fields on SO(3)

In the present era, there is a growing interest in modeling data not only with scalar values but also with more sophisticated algebraic structures. An important example is spin spherical random fields, that can be defined as random sections of a bundle of the 2-sphere. These fields manifest in both gravitational lensing data and Cosmic Microwave Background polarization data, which can be seen as vectors on the complex plane.

Motivated by studying anisotropies and divergence from Gaussianity of the latter, we study the excursion sets of their real part by means of their Lipschitz-Killing curvatures, that are geometric functionals such as, in dimension 3, the volume, the surface area, the cross-sectional diameter and the Euler characteristic. Without requiring the isotropy of the field, it is possible to compute explicitly the expectation of such functionals in terms of the spin parameter and the level of the excursion.
The talk is based on the joint work "Expected Lipschitz-Killing curvatures for spin random fields and other non-isotropic fields" (2024) with M. Stecconi (submitted, arXiv:2406.04850).

- **Ioan Scheffel, University of Stuttgart, (Germany)**

Stationary Maxima of non-stationary Particle Systems

We consider particle systems of the form

$$\mathfrak{P}(U,V) \;:=\; \{U^{(i)} \cdot V_t^{(i)} \mid i \in \mathbb{N}\},$$

where $(U^{(i)})$ are points of a Poisson point process (PPP) and $(V_t^{(i)})$ are independent and identically distributed samples of a stochastic process $V = (V_t)$. It is well known that $\mathfrak{P}(U,V)$ can be stationary even if V is non-stationary. For example, for a Markov process V the particle system $\mathfrak{P}(U,V)$ is stationary if and only if the intensity measure of the PPP is invariant with respect to V. We study maxima of particle systems, that is

$$\max_{i\in\mathbb{N}} U^{(i)} V_t^{(i)}.$$

Can non-stationary particle systems have stationary maxima? We build an example using stochastic differential equations and extreme value theory.

- **Michael Staněk, Ulm University (Germany)**

Vague and basic convergence of signed measures
We study the recently introduced concept of basic convergence. We discover that it can be used to characterize vague convergence of signed measures in terms of convergence of its distribution functions.

- **Issey Sukeda, The University of Tokyo (Japan)**

Torus graph modelling for EEG analysis
Identifying phase coupling among electroencephalography (EEG) signals recorded from multiple electrodes helps us understand the underlying dependence structure in the brain. From a statistical perspective, these signals represent multi-dimensional angular measurements that are correlated with each other, and can be effectively modeled using the torus graph model designed for circular random variables. Regularization is typically employed to achieve a sparse network structure; however, some degree of arbitrariness remains. To enhance the validity of this approach, we propose using regularized score

matching estimation for the torus graph based on information criteria. In numerical simulations, our method successfully recovers the true dependence structure from the data. Additionally, we present analyses of real EEG data.

- **Sebastian Zeng, University of Salzburg (Austria)**

Neural Persistence Dynamics

We consider the problem of learning the dynamics in the topology of time-evolving point clouds, the prevalent spatiotemporal model for systems exhibiting collective behaviour, such as swarms of insects and birds or particles in physics. In such systems, patterns emerge from (local) interactions among self-propelled entities. While several well-understood governing equations for motion and interaction exist, they are difficult to fit to data due to the often-large number of entities and missing correspondences between the observation times, which may also not be equidistant. To evade such confounding factors, we investigate collective behaviour from a ***topological perspective***, but instead of summarizing entire observation sequences (as in prior work), we propose learning a latent dynamical model from topological features ***per time point***. The latter is then used to formulate a downstream regression task to predict the parametrization of some a priori specified governing equation. We implement this idea based on a latent ODE learned from vectorized (static) persistence diagrams and show that this modeling choice is justified by a combination of recent stability results for persistent homology. Various (ablation) experiments not only demonstrate the relevance of each individual model component, but provide compelling empirical evidence that our proposed model – ***neural persistence dynamics*** – substantially outperforms the state-of-the-art across a diverse set of parameter regression tasks.