Implicit linear difference equation over a non-Archimedean ring

A.B. Goncharuk

Let us consider the difference equation

$$a_m x_{n+m} + \ldots + a_1 x_{n+1} + a_0 x_n = f_n,\ n = 0, 1, 2, \ldots$$

We are looking for the sequence $\{x_n\}$ that satisfies this equation.

Note that for any field F, if $a_m, \ldots, a_0 \in F$ and $f_n \in F$ for any n, it surely has infinitely many solutions over F – one for each initial value $x_0, x_1, \ldots, x_m$. The interesting question appears for some ring K in the case that a_m is non-invertible. We are looking for the solution that belongs to K supposed all the coefficients a_i and the nonhomogeneity f_n belong to K . Is there at least one solution in this ring? How many solutions there are? How to find this solution?

The talk is devoted to this question in the case K is a valuation ring of some field with a non-Archimedean valuation. It is an important case, because finding a solution of our equation considered over two important rings – the ring of integers and the ring of polynomials – in many cases reduces to finding the solution in this class of non-Archimedean rings.