# Statistik früher kosmischer Strukturen und die Größe des Universum


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Institut für Stochastik,
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# Outline

cosmic structures
→ early cosmic structures
→ cosmic microwave background (CMB)

fluctuations in the CMB
→ statistical properties
→ appropriate measures

finite universes
→ motivation, properties and effects
→ the size of the Universe

concluding remarks

# cosmic structures

## the Milkyway

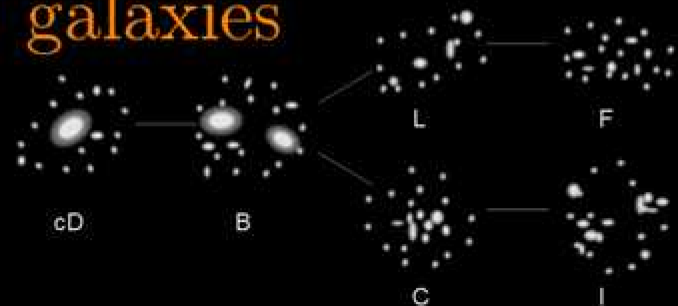


$$
\begin{aligned}
1\ Ly &= 9.5 \times 10^{15} m \\
radius &= 50000\ Ly \\
\#\ stars &= 200\ billion
\end{aligned}
$$

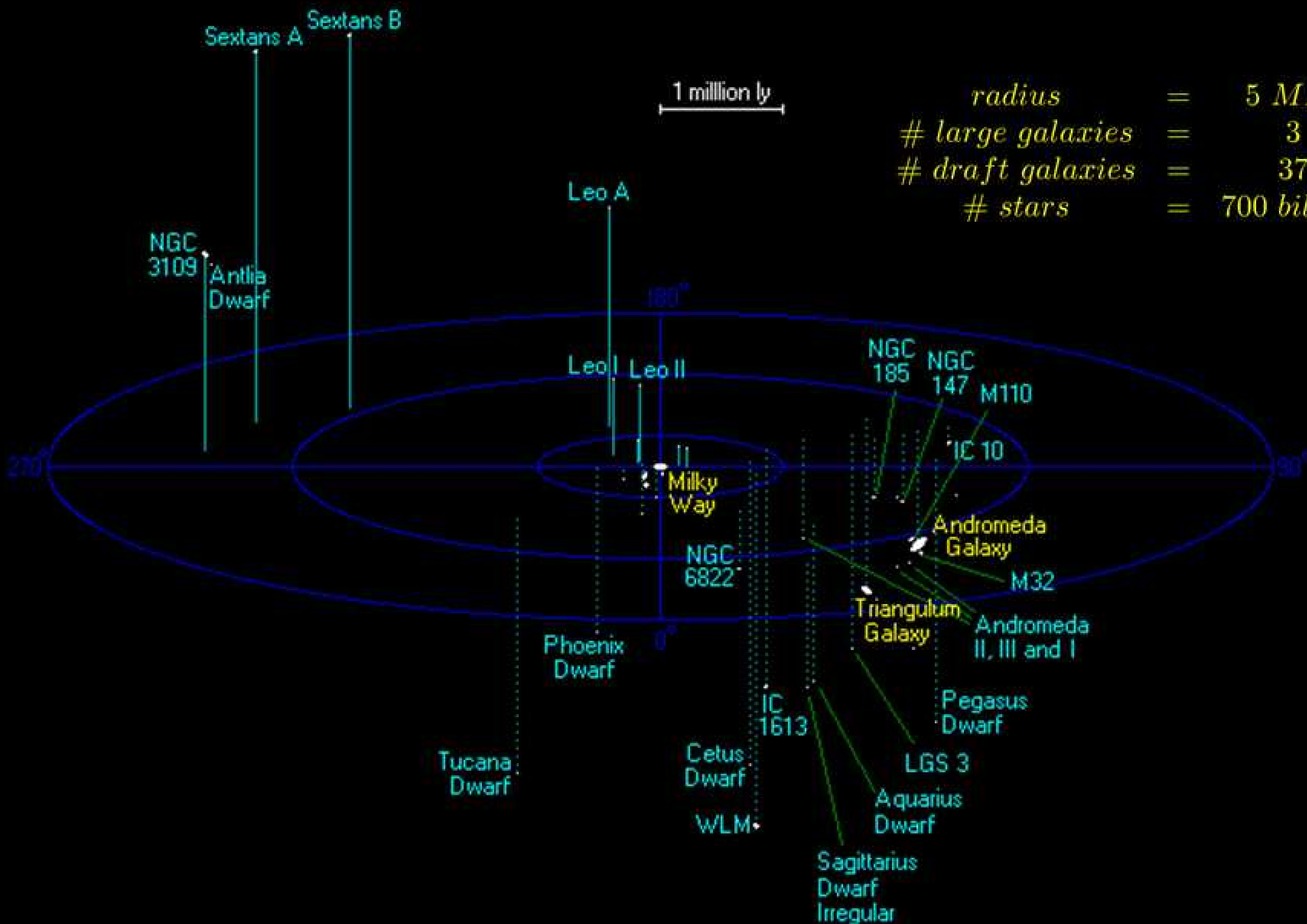
## types of galaxies


## Andromeda (M31)

## draft galaxies



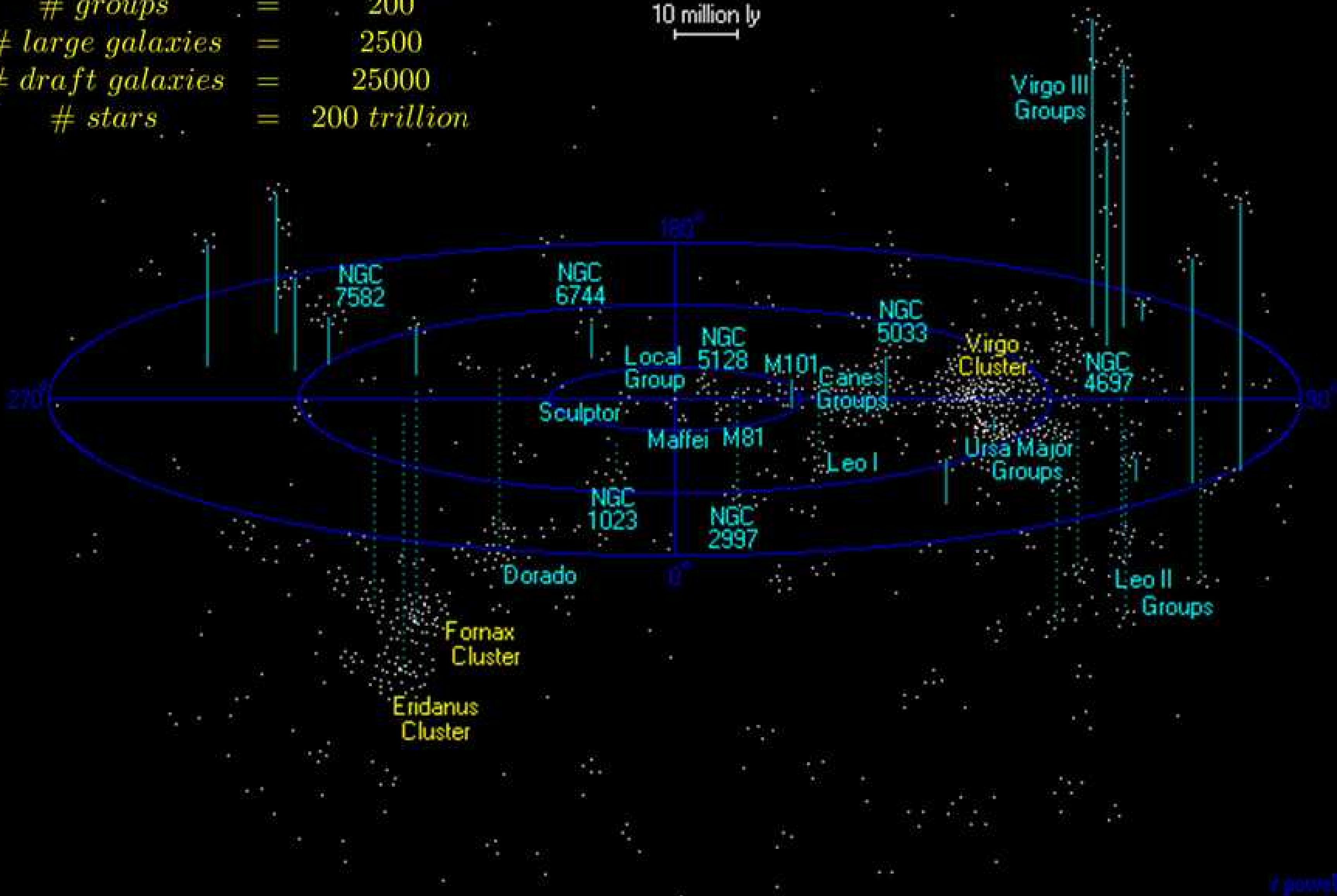
# atlas of our neightbourhood: the local group (galaxies)



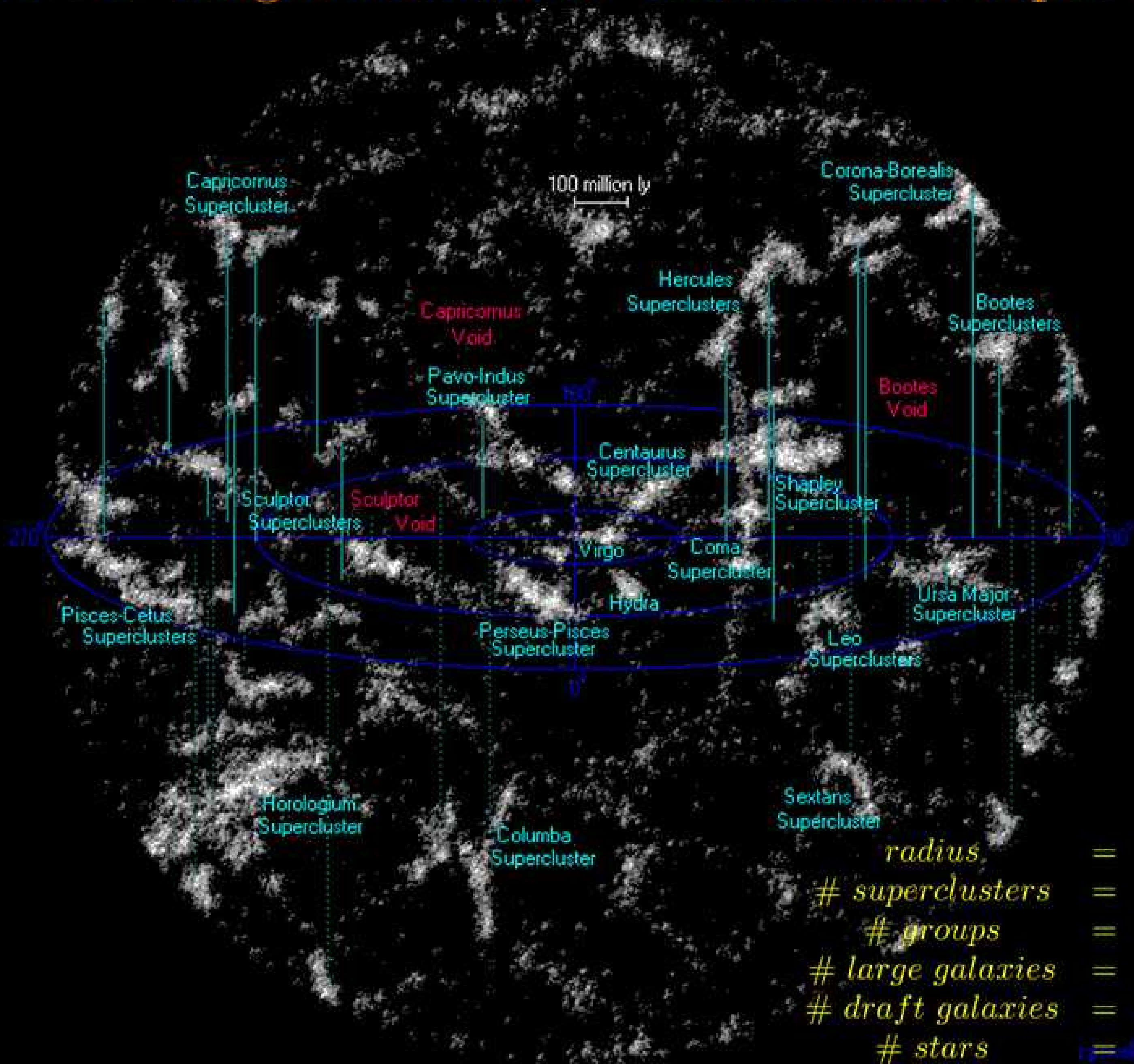

| | | |
|---|---|---|
| *radius* | = | $5\ MLy$ |
| *# large galaxies* | = | 3 |
| *# draft galaxies* | = | 37 |
| *# stars* | = | *700 billion* |

# atlas of our neightbourhood: virgo cluster (clusters)

| | | |
|---|---|---|
| *radius* | = | 100 $MLy$ |
| # *groups* | = | 200 |
| # *large galaxies* | = | 2500 |
| # *draft galaxies* | = | 25000 |
| # *stars* | = | 200 *trillion* |

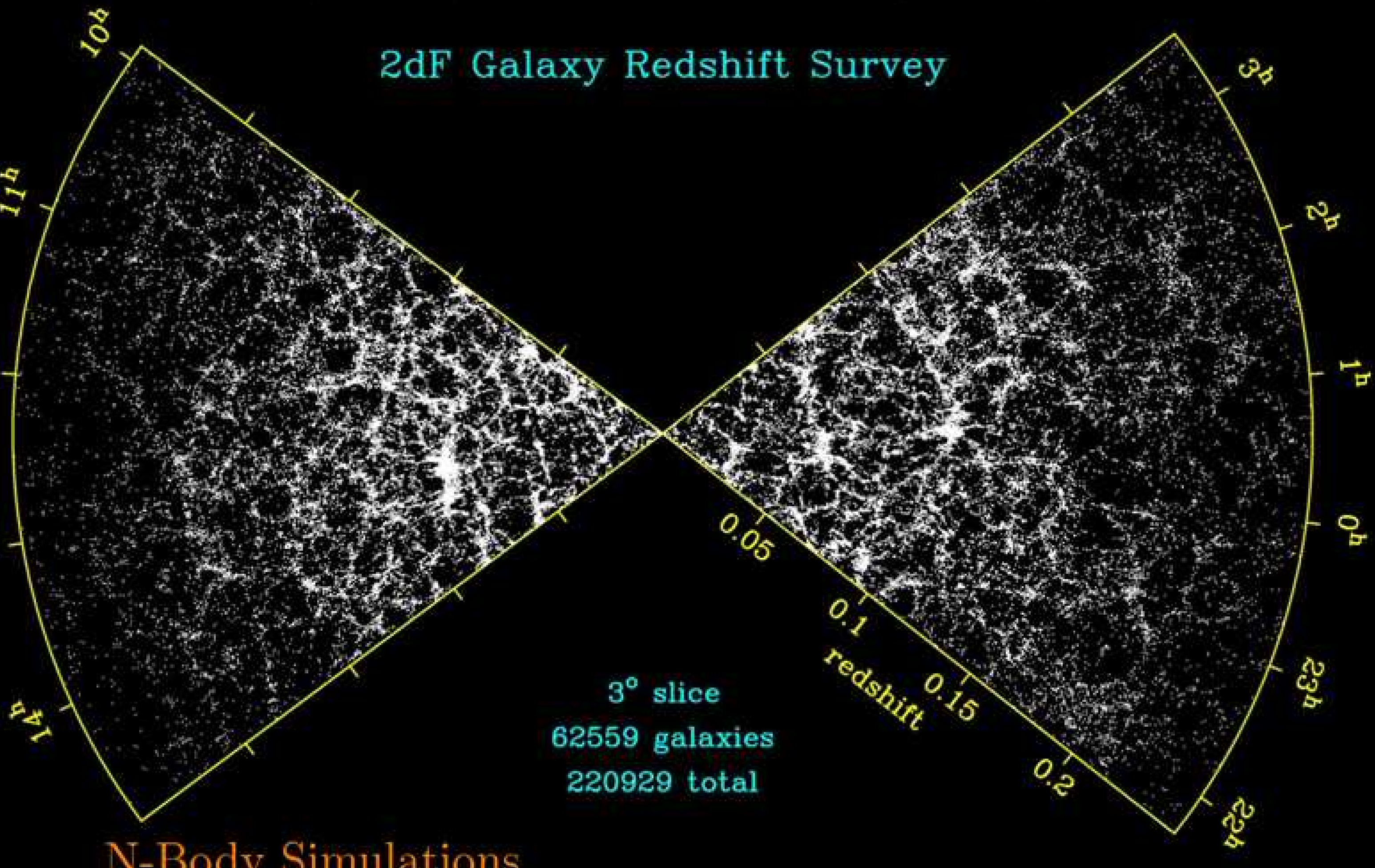


# atlas of our neighbourhood: clusters and super clusters

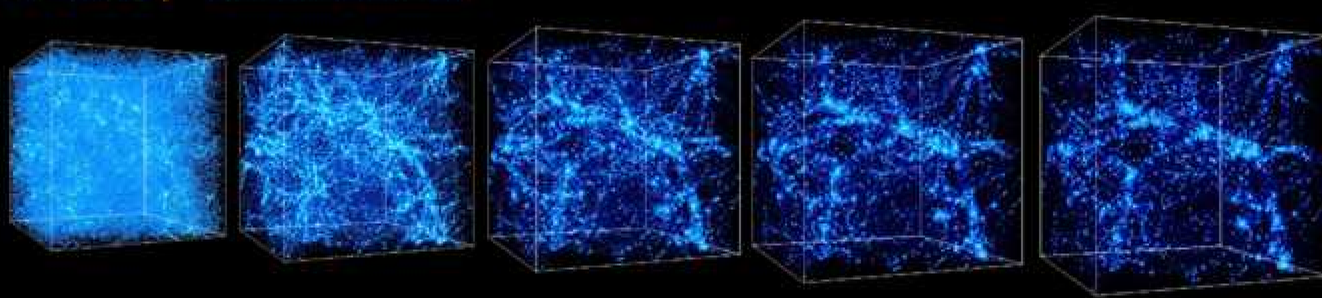



| | | |
|---|---|---|
| *radius* | = | $1\ GLy$ |
| *# superclusters* | = | 100 |
| *# groups* | = | 240000 |
| *# large galaxies* | = | *3 million* |
| *# draft galaxies* | = | *30 million* |
| *# stars* | = | *250000 trillion* |

# observations of galaxy clusters and galaxy superclusters

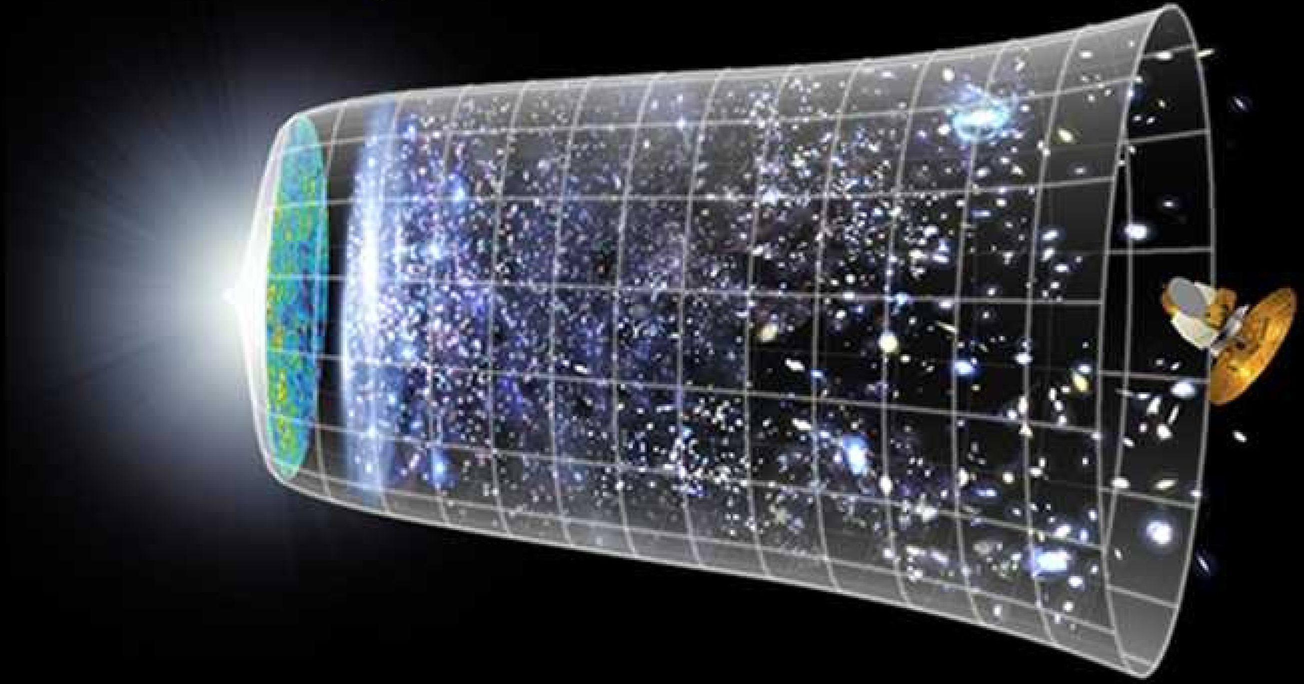



## N-Body Simulations



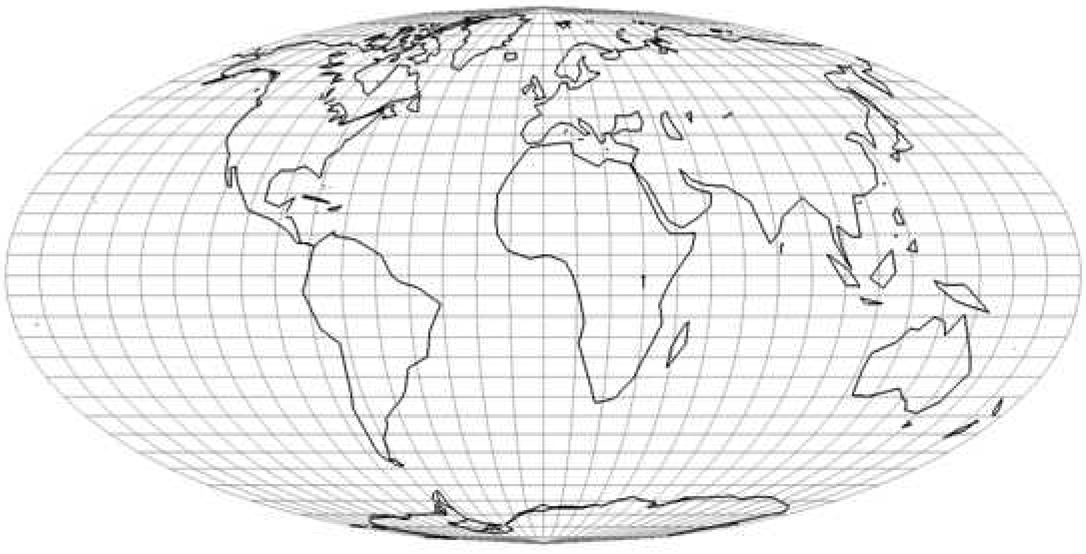
# evolution of cosmic structures and the Universe

$$\mathbb{M}_{Universe} = \mathbb{R}_{time} \times \mathbb{M}_{space}$$



# cosmic microwave background (CMB)

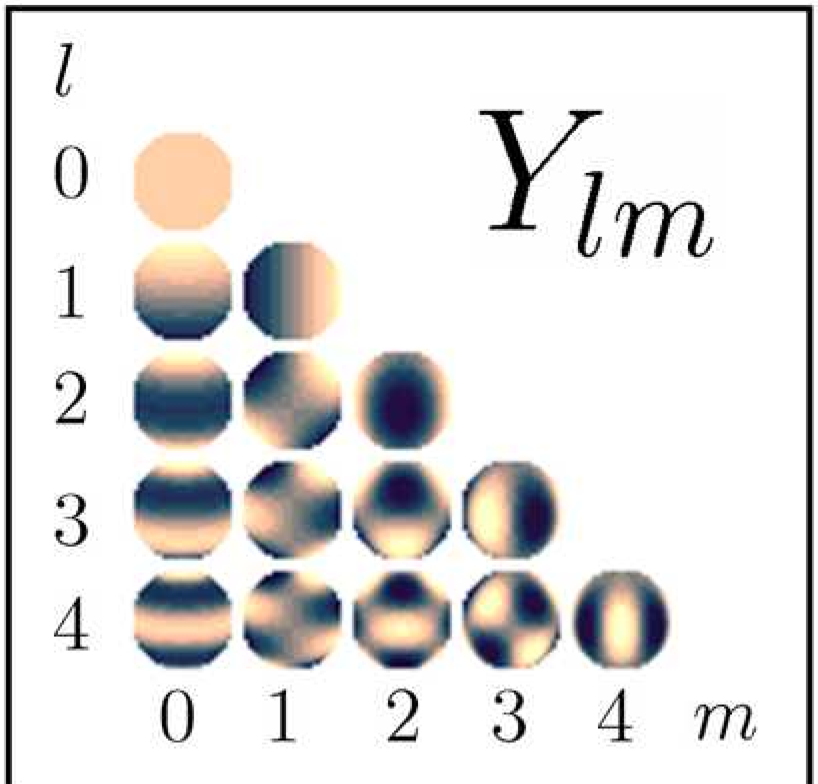
Mollweide projektion



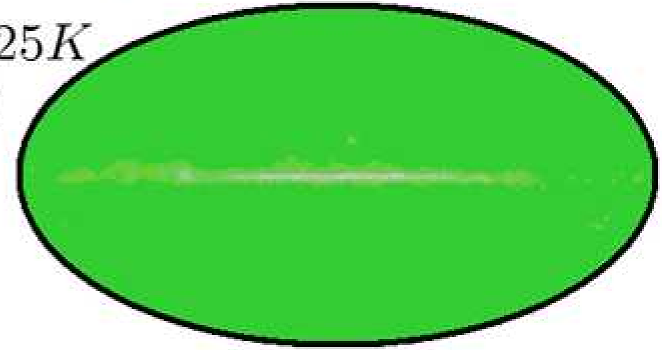
# cosmic microwave background (CMB)

expansion in spherical harmonics

$$\Delta T(\theta,\phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi)$$

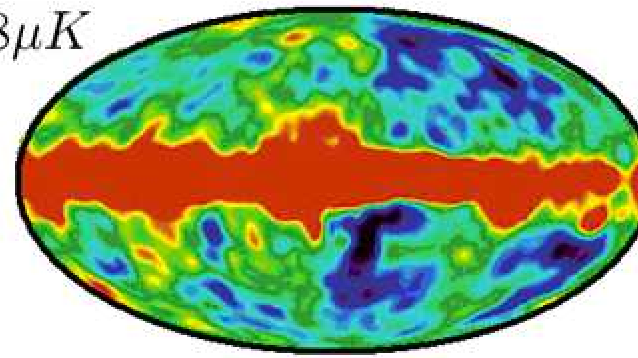



$T = 2.725K$
$l = 0$



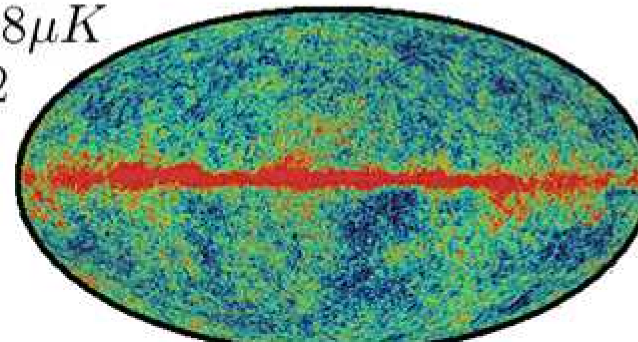
[a] Penzias and Wilson (1965)

$\Delta T = 18\mu K$
$l \geq 2$



[b] COBE (1992)

$\Delta T = 18\mu K$
$l \geq 2$



[c] WMAP (2003)

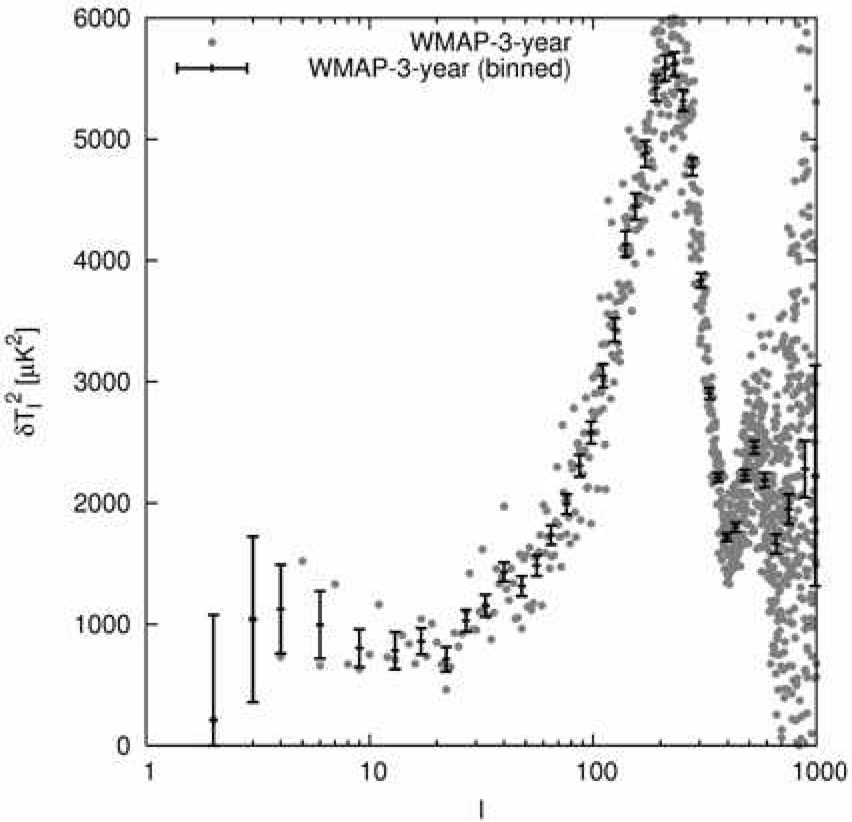
# cosmic microwave background (CMB)

expansion in spherical harmonics

$$\Delta T(\theta,\phi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi)$$

multipole moment: $C_l := \frac{1}{2l+1}\sum_{m=-l}^{l} |a_{lm}|^2$

power spectrum: $\delta T_l^2 := \frac{l(l+1)}{2\pi} C_l$

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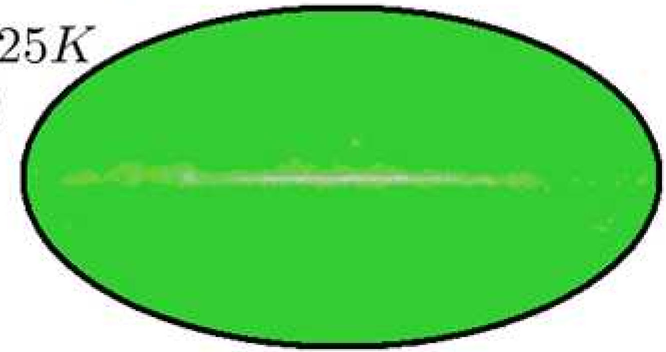

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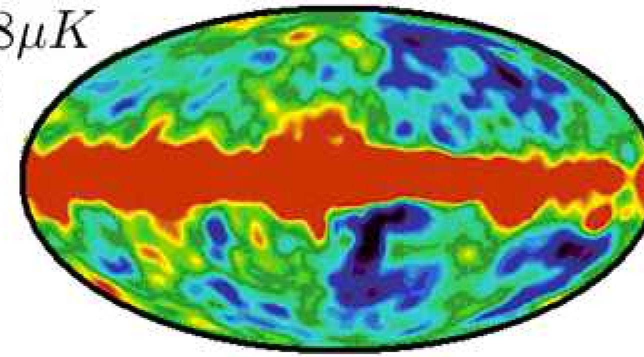

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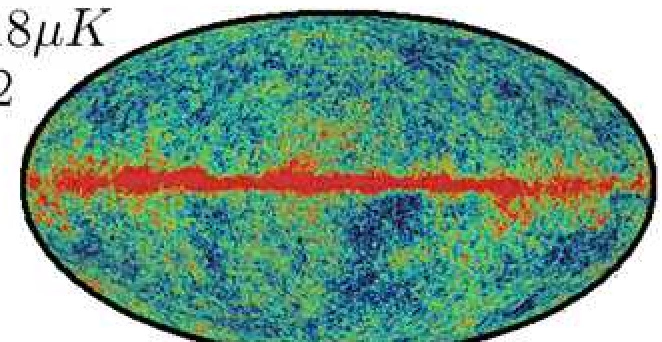

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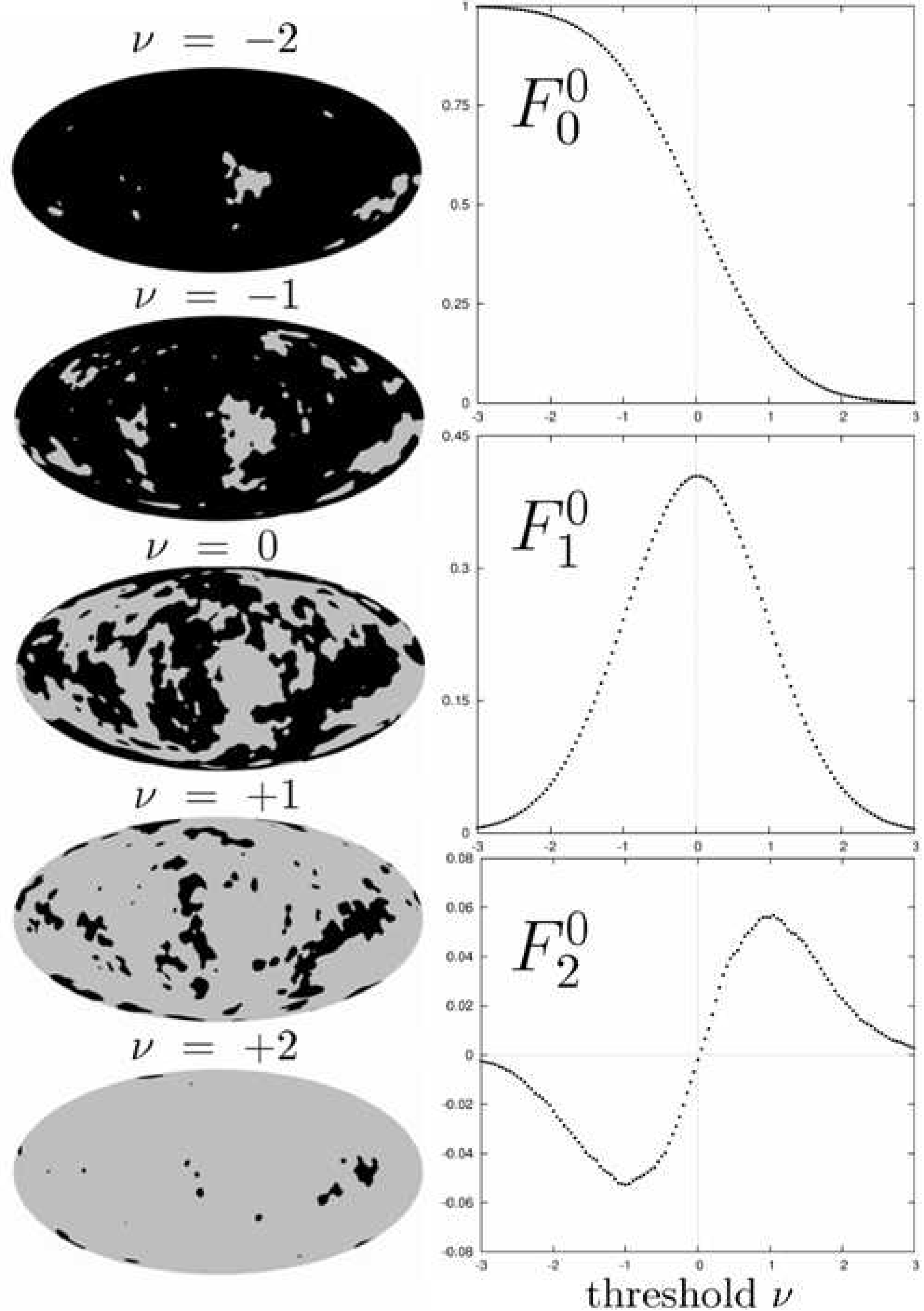



# Statistics of the CMB

Excursion Set

$$Q_\nu = \{x \in \mathbb{S}^2 | \frac{\Delta T(x)}{\sigma} \geq \nu\}$$

$$\partial Q_\nu = \{x \in \mathbb{S}^2 | \frac{\Delta T(x)}{\sigma} = \nu\}$$

Variance of the field

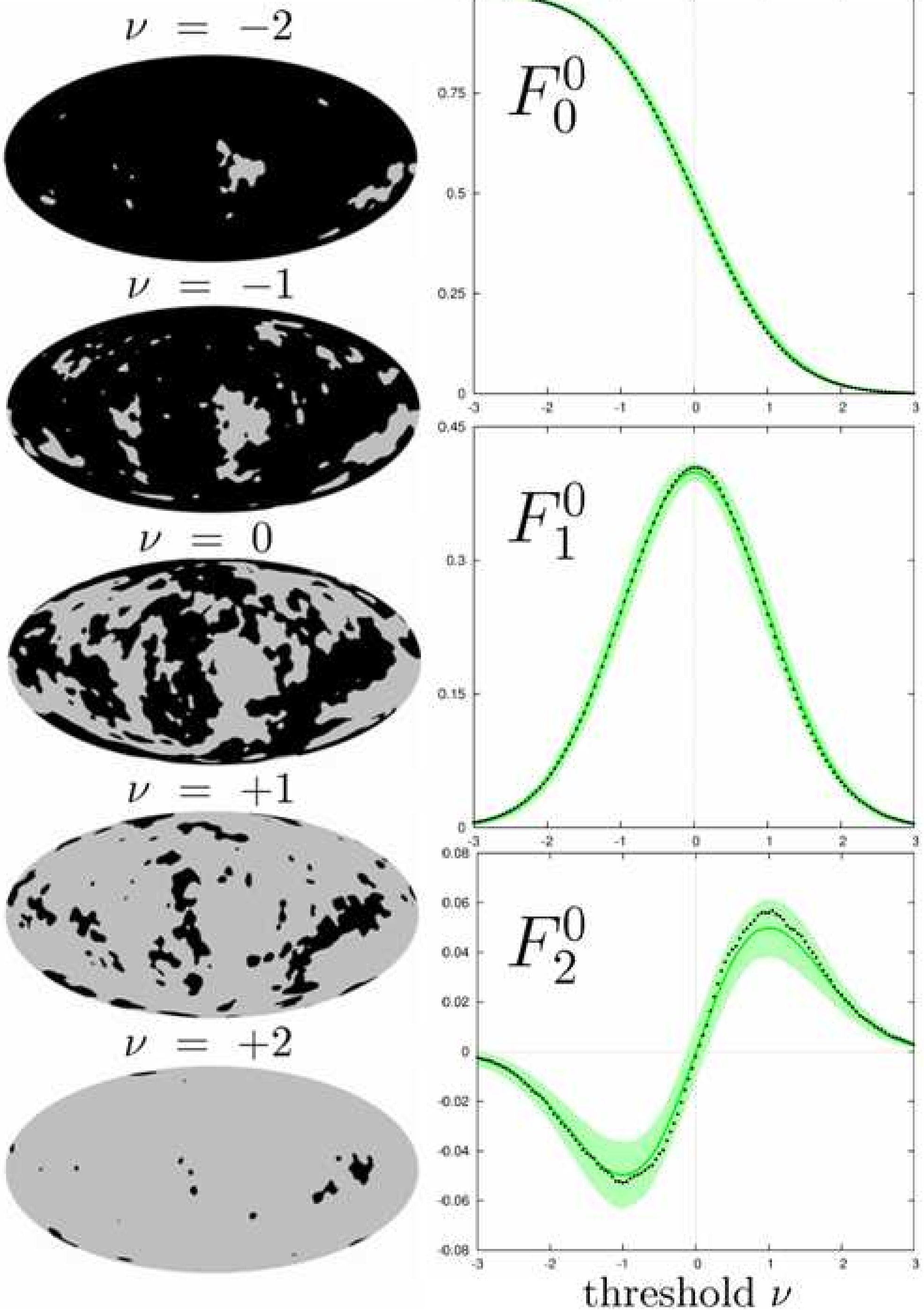
$$\sigma^2 = \sum_{l=2}^{\infty} (2l+1) C_l = 4\pi C(0)$$

Minkowski Functionals

$$F_0^0(Q_\nu) \propto \int_{Q_\nu} da$$

$$F_1^0(Q_\nu) \propto \int_{\partial Q_\nu} dl$$

$$F_2^0(Q_\nu) \propto \int_{\partial Q_\nu} dl \ \kappa$$

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Variance of the field

$$\sigma^2 = \sum_{l=2}^{\infty}(2l+1)C_l = 4\pi C(0)$$

Minkowski Functionals

$$F_0^0(Q_\nu) \propto \int_{Q_\nu} da \quad \propto \frac{1}{2} - \frac{1}{2}\Phi\left(\frac{\nu}{\sqrt{2}}\right)$$

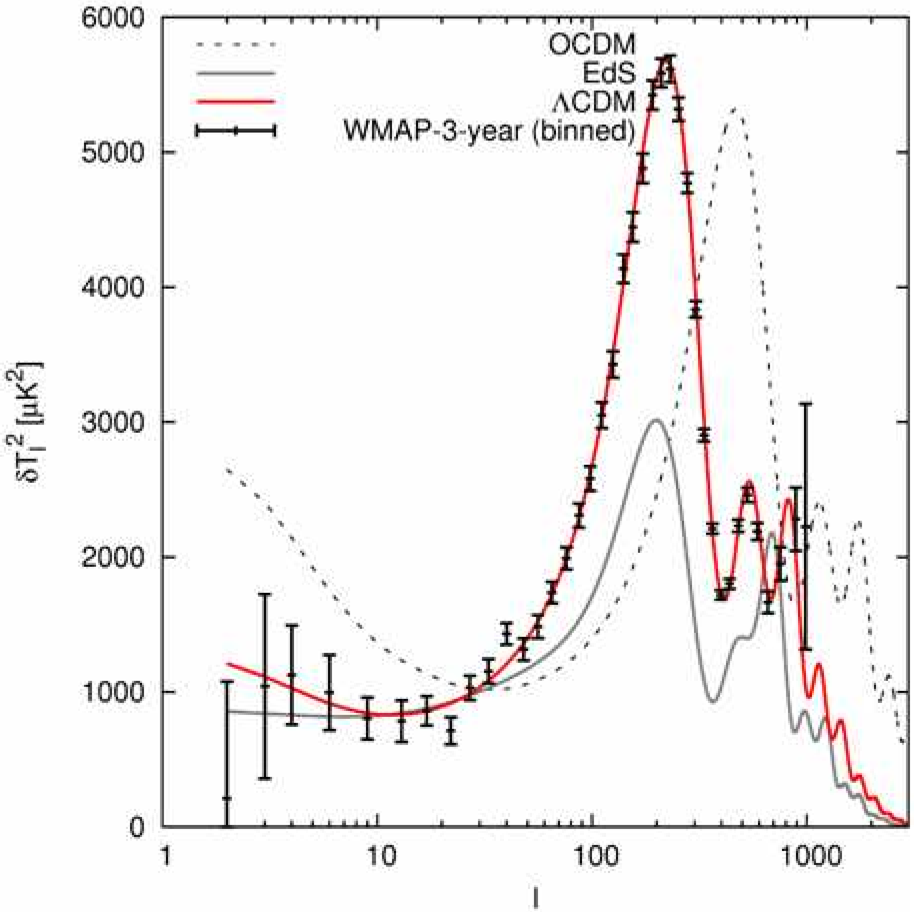
$$F_1^0(Q_\nu) \propto \int_{\partial Q_\nu} dl \quad \propto \exp\left(-\frac{\nu^2}{2}\right)$$

$$F_2^0(Q_\nu) \propto \int_{\partial Q_\nu} dl\ \kappa \quad \propto \nu \exp\left(-\frac{\nu^2}{2}\right)$$

gaussian random field

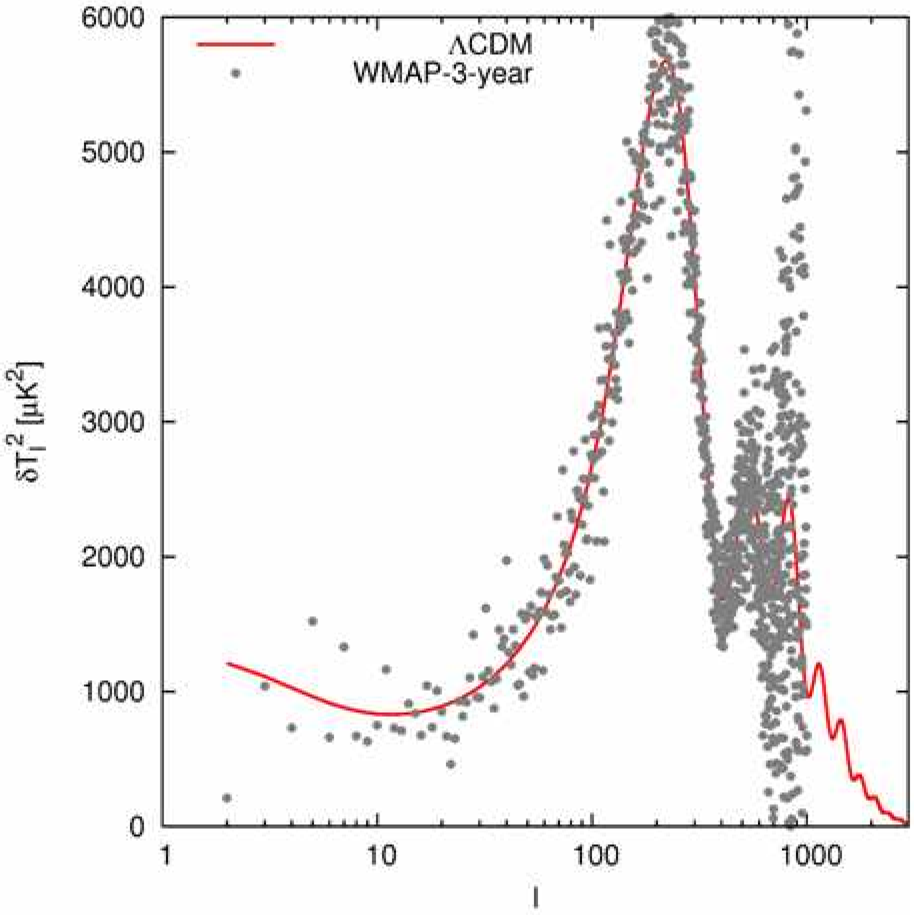
# Simulations

power spectrum

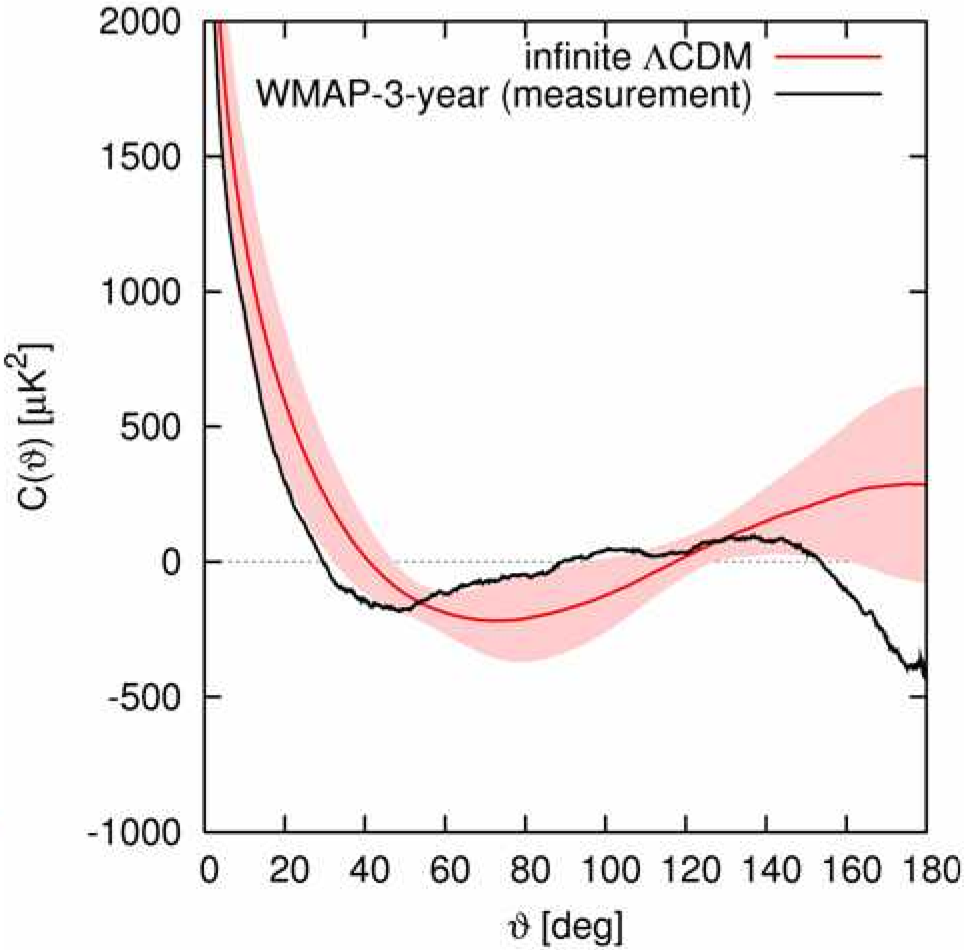


# Simulations

power spectrum




2-Point-Correlation-Function



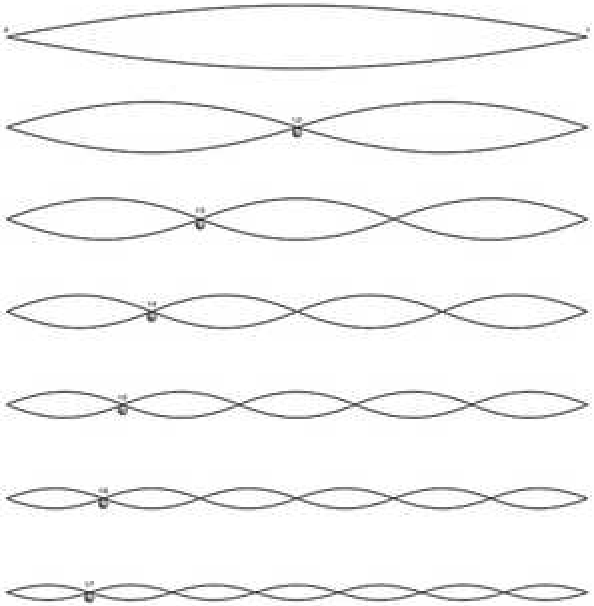

2-Point-Correlation-Function: $C(\vartheta) := \langle \Delta T(\theta_1, \phi_1) \Delta T(\theta_2, \phi_2) \rangle_{\vec{n}_1 \cdot \vec{n}_2 = \cos\vartheta}$

transformation: $C(\vartheta) = \sum_{l=0}^{\infty} C_l \frac{2l+1}{4\pi} P_l(\cos\vartheta),\ C_l = 2\pi \int_{-1}^{1} d\cos\vartheta\ C(\vartheta)\ P_l(\cos\vartheta)$

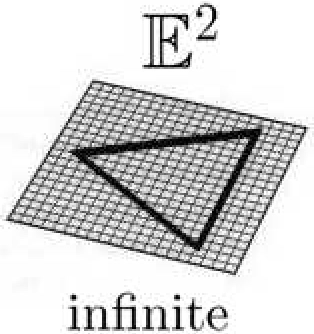
# finite universes

## Motivation

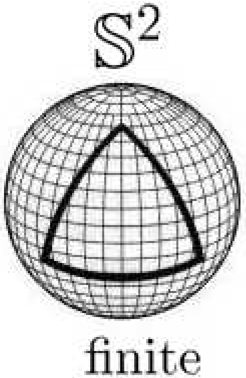
overtones on a string



General Relativity: possible Friedmann-Lemaitre universes illustrated in 2 dimensions





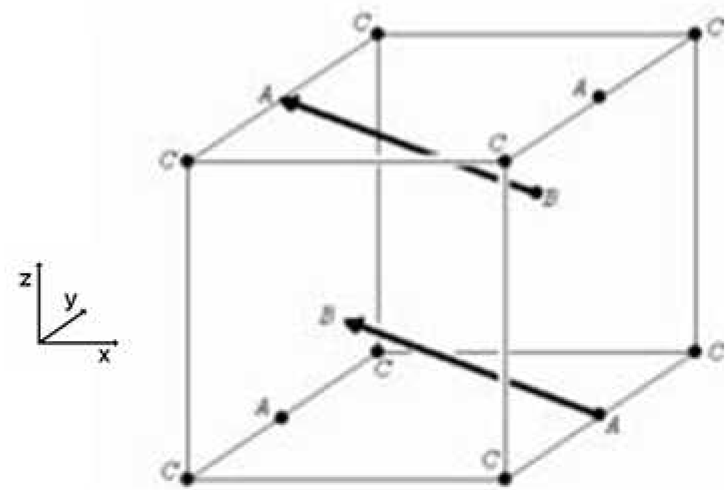


# finite universes with nontrivial topology

Pacman (1980)


flat 3-Torus

$$\mathbb{T}^3 := \mathbb{E}^3/\Gamma,\ \gamma \in \Gamma, \gamma : \vec{x} \to \vec{x} + \vec{T},$$
$$\vec{T} = L_x n_x \hat{e}_x + L_y n_y \hat{e}_y + L_z n_z \hat{e}_z,$$



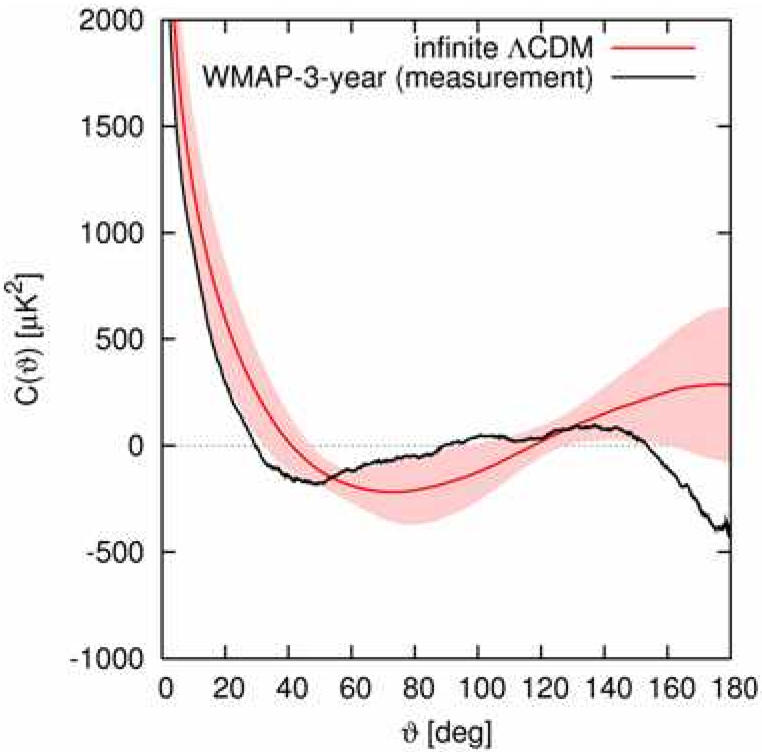

$$(n_x, n_y, n_z) \in \mathbb{Z}^3,\ \exp[i\vec{k}\vec{x}] = \exp[i\vec{k}(\vec{x} + \vec{T})],$$
$$\vec{k} = 2\pi(\frac{n_x}{L_x}\hat{e}_x + \frac{n_y}{L_y}\hat{e}_y + \frac{n_z}{L_z}\hat{e}_z),$$

$$k_{\min} = 2\pi \min\{\frac{1}{L_x}, \frac{1}{L_y}, \frac{1}{L_z}\},$$
$$\lambda_{\max} = \max\{L_x, L_y, L_z\}$$

# finite universes as a flat 3-Torus

first infinite $\Lambda CDM$

2-Point-Correlation-Function



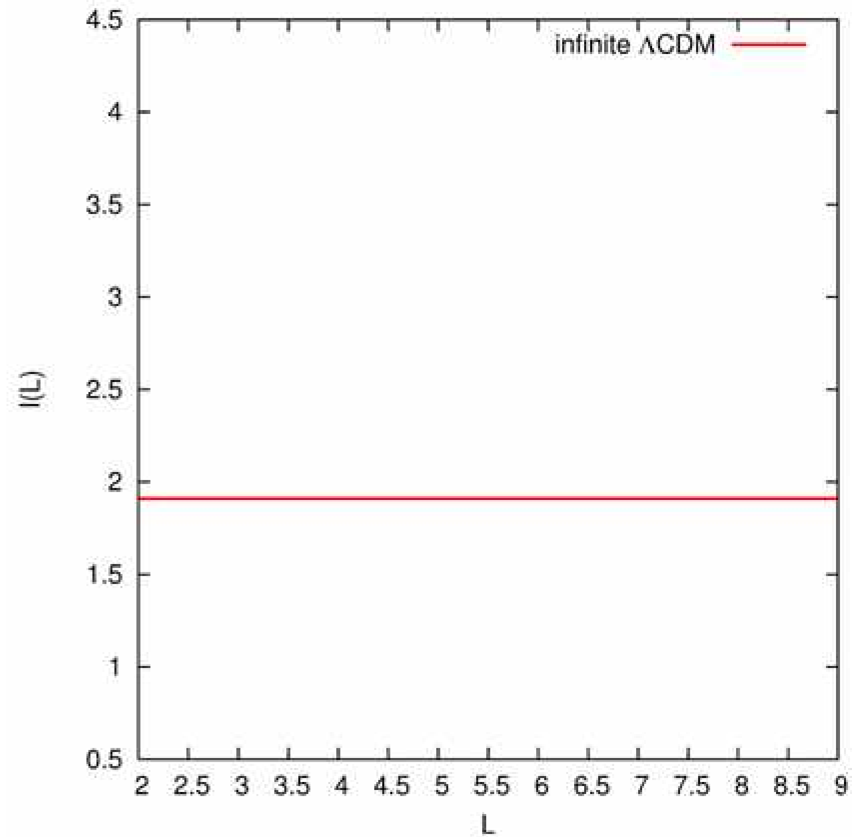

flat 3-Torus

$$L_x \equiv L_y \equiv L_z \equiv L \times L_H, \; L \in \mathbb{R}$$

$$L_H := \frac{c}{H_0} = \frac{3000}{h} Mpc \approx 14 \; GLy \approx 4.3 \times 10^{20} m$$

integrated weighted correlation difference

$$I := \int_{-1}^{1} d\cos\vartheta \, \frac{[C^{model}(\vartheta) - C^{obs}(\vartheta)]^2}{Var[C^{model}(\vartheta)]}$$



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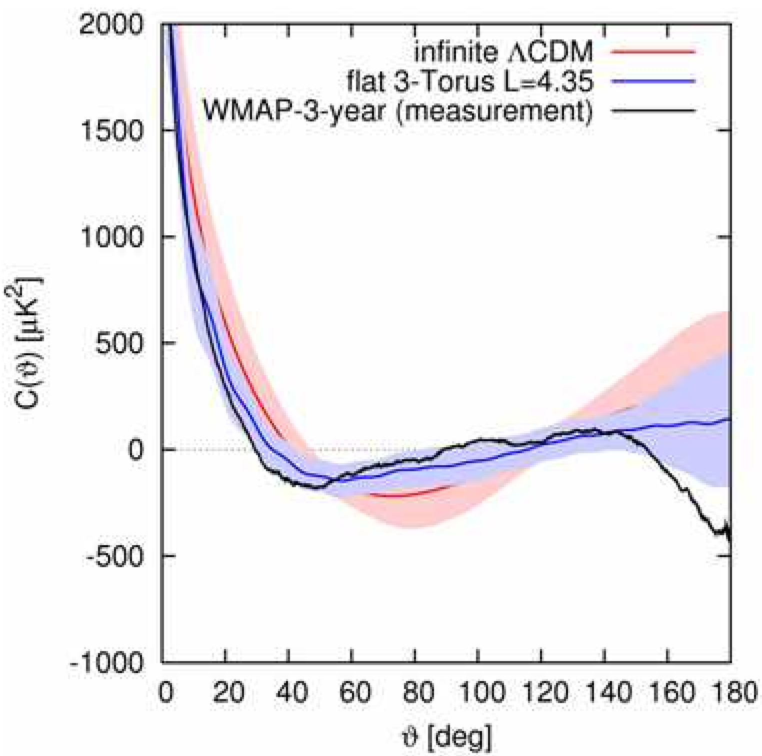


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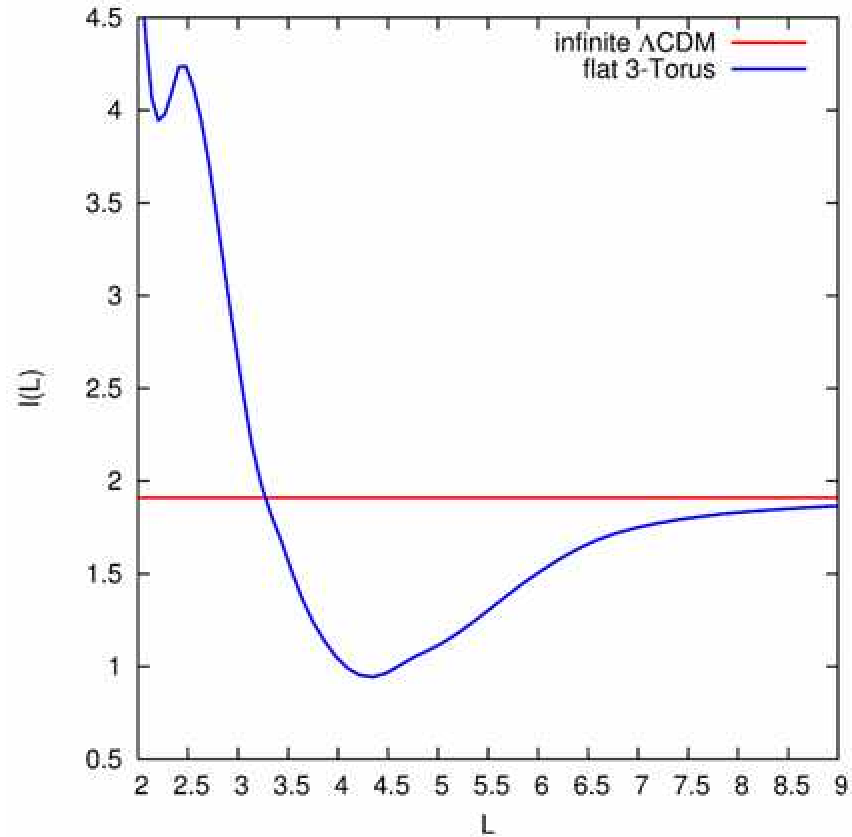


# concluding remarks

statistics of the CMB

- global statistics of the CMB
- local deviations and artefacts
- new and better methods

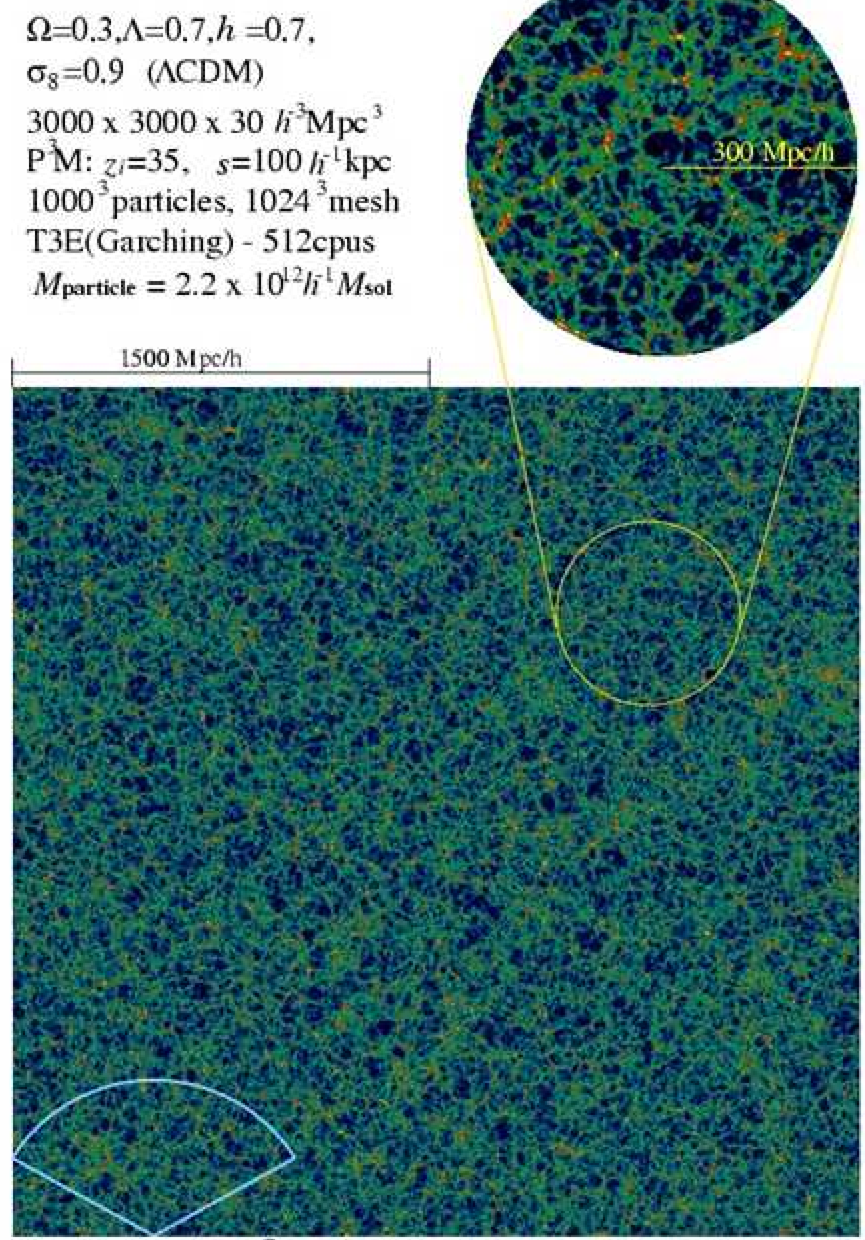
geometry and topology of the universe

- other spaceforms
- independent methods

experiments in the future

- new data from WMAP
- LHC at Cern 2008
- PLANCK by ESA 2008

The Hubble Volume Simulation




$(4.35 \times L_H)^3 : \#stars \approx 13\ billion\ trillion$